McGill University, Montreal
GEOG 321 - Climatic Environments
Knox

Answers to Study Questions - Topic 23

1. The production rate of mechanical turbulence is described as:

$$-\overline{u'w'}\frac{\Delta \overline{u}}{\Delta z}$$

Both u' and w' have units of $\mathrm{m\,s^{-1}}$, so their (average) product $\overline{u'w'}$ has the unit of $\mathrm{m^2\,s^{-2}}$.

$\Delta\overline{u}$ has units of $\mathrm{m\,s^{-1}}$ and Δz is given in m. So the wind gradient $\Delta\overline{u}/\Delta z$ has units of $\mathrm{m\,s^{-1}\,m^{-1}} = \mathrm{s^{-1}}$.

The combined mechanical production term then must have units of $\mathrm{m^2\,s^{-2} s^{-1}} = \underline{\mathrm{m^2\,s^{-3}}}$.

The production rate of thermal turbulence is described as:

$$\frac{g}{\overline{T}}\overline{w'T'}$$

g is the acceleration due to gravity and describes the velocity increase per time with the units of $(\mathrm{m\,s^{-1}})\,\mathrm{s^{-1}} = \mathrm{m\,s^{-2}}$. $\overline{T}$ is the absolute temperature in K. So the term $g/\overline{T}$ has the units $\mathrm{m\,s^{-2}\,K^{-1}}$.

w' has units of $\mathrm{m\,s^{-1}}$, and T' has units of K, so their (average) product $\overline{w'T'}$ has the unit of $\mathrm{m\,s^{-1}\,K}$.

The combined thermal production term then must have units of $\mathrm{m\,s^{-2}}$ $\mathrm{K^{-1} m\,s^{-1}\,K} = \underline{\mathrm{m^2\,s^{-3}}}$ as the Kelvins cancel out.

2. TKE is an energy in Joules (J). The SI unit J can be also written as (see e.g. https://en.wikipedia.org/wiki/Joule)

$$\mathrm{J} = \mathrm{kg\,m^2\,s^{-2}}$$

We usually express TKE per unit mass (i.e. $\mathrm{kg^{-1}}$, see Lecture 19), so the units of TKE per unit mass would be $\mathrm{kg\,m^2\,s^{-2}\,kg^{-1}} = \mathrm{m^2\,s^{-2}}$.

The amount of TKE per unit mass produced per time ($\mathrm{s^{-1}}$, production rate) is then $\mathrm{m^2\,s^{-2} s^{-1}}$ hence $\underline{\mathrm{m^2\,s^{-3}}}$, the same as the above terms. So both terms describe the rate of TKE per unit mass and per unit time produced.

3. The mechanical production rate is:

$$-\overline{u'w'}\frac{\Delta\overline{u}}{\Delta z} = - -0.52\,\mathrm{m^2\,s^{-2}} \times 0.07\,\mathrm{s^{-1}} = \underline{0.0364\,\mathrm{m^2\,s^{-3}}}$$

The thermal production rate is:

$$\frac{g}{\overline{T}}\overline{w'T'} = \frac{9.81\mathrm{m\,s^{-2}}}{304.1\mathrm{K}} \times 0.30\,\mathrm{K\,m\,s^{-1}} = \underline{0.0097\,\mathrm{m^2\,s^{-3}}}$$

The total production rate is the sum of thermal and mechanical production rates:

$$0.0364\,\mathrm{m^2\,s^{-3}} + 0.0097\,\mathrm{m^2\,s^{-3}} = \underline{0.0461\,\mathrm{m^2\,s^{-3}}}$$

4. The Richardson flux number (Rf) is the ratio of thermal to (minus) mechanical production rate, hence inserting values from Question 3:

$$Rf = \frac{\frac{g}{\overline{T}}\overline{w'T'}}{\overline{u'w'}\frac{\Delta\overline{u}}{\Delta z}} = \frac{0.0097\,\mathrm{m^2\,s^{-3}}}{-0.0364\,\mathrm{m^2\,s^{-3}}} = \underline{-0.2665}$$

Note that Rf is a dimensionless number and has no units.

5. The result fulfills $-1/3 < -0.2665 < 1/3$, hence falls into a the turbulence regime of 'forced convection'. It is a dynamically slightly unstable situation (i.e. $Rf < 0$).

6. The height in the surface layer at where the mechanical production rate and the thermal production rate are equal is equal to (minus) the Obukhov length L. The Obukhov length L is defined as:

$$L = -\frac{\overline{T}\,u_*^3}{k\,g\,\overline{w'T'}}$$

Here, k is the von Karman constant (0.41), also g is a constant. We have $\overline{w'T'}$ and $\overline{T}$, but we first need to calculate u_*, the friction velocity in $\mathrm{m\,s^{-1}}$:

$$u_* = \sqrt{-\overline{u'w'}} = \sqrt{- -0.52\,\mathrm{m^2\,s^{-2}}} = 0.721\,\mathrm{m\,s^{-1}}$$

Enter into the equation for the Obukhov length L:

$$L = -\frac{\overline{T}\,u_*^3}{k\,g\,\overline{w'T'}} = -\frac{304.1\mathrm{K} \times (0.721\,\mathrm{m\,s^{-1}})^3}{0.41 \times 9.81\mathrm{m\,s^{-2}} \times 0.30\,\mathrm{K\,m\,s^{-1}}} = -94.5034\,\mathrm{m}$$

So at a height of minus L, i.e. at 94 m, thermal and mechanical production rates are expected to be equal.

7. The stability parameter ζ is defined as

$$\zeta = \frac{z}{L} = \frac{10\text{m}}{-94.50\,\text{m}} = \underline{-0.106}$$

8. A dynamic stability of $\zeta < 0$ is dynamically slightly unstable. This matches the $Rf < 0$ situation found above.