Answers to Study Questions - Topic 22

January 2, 2024

1. Read the air density ρ_a from Oke 'Boundary Layer Climates' Table A3.1.

$$
\begin{aligned}
Q_H &= C_a \overline{w'T'} = \rho_a\, c_p\, \overline{w'T'} \\
&= 1.230\,\text{kg m}^{-3} \times 1010\,\text{J kg}^{-1}\,\text{K}^{-1} \times -0.031\,\text{m s}^{-1}\,\text{K} \\
&= -38.5\,\text{J m}^{-2}\,\text{s}^{-1} = \underline{-38.5\,\text{W m}^{-2}}
\end{aligned}
$$

This is likely a night-time situation, when the surface is cold and that atmosphere is warmer. The profile of potential temperature is likely increasing with height. Excess sensible heat (i.e. $\rho_a c_p T$) is transported from higher levels of the atmosphere down to the layers close to the surface, as indicated by the negative sign of Q_H.

2. Use the latent heat of vaporization for water, L_v, form Oke 'Boundary Layer Climates' Table A3.1.

$$
\begin{aligned}
Q_E &= L_v \overline{w'\rho_v'} \\
&= 2.432 \times 10^6\,\text{J kg}^{-1} \times 1.73 \times 10^{-4}\,\text{kg m}^{-2}\,\text{s}^{-1} \\
&= 420\,\text{J m}^{-2}\,\text{s}^{-1} = \underline{420\,\text{W m}^{-2}}
\end{aligned}
$$

3. The bowen ratio is $\beta = Q_H/Q_E$. Analogous to Question 1, calculate Q_H:

$$
\begin{aligned}
Q_H &= \rho_a\, c_p\, \overline{w'T'} \\
&= 1.188\,\text{kg m}^{-3} \times 1010\,\text{J kg}^{-1}\,\text{K}^{-1} \times 0.121\,\text{m s}^{-1}\,\text{K} \\
&= 145.1\,\text{W m}^{-2}
\end{aligned}
$$

Analogous to Question 2, calculate Q_E:

$$
\begin{aligned}
Q_E &= L_v \overline{w'\rho_v'} \\
&= 2.453 \times 10^6\,\text{J kg}^{-1} \times 1.21 \times 10^{-4}\,\text{kg m}^{-2}\,\text{s}^{-1} \\
&= 296.8\,\text{W m}^{-2}
\end{aligned}
$$

Hence

$$\beta = \frac{Q_H}{Q_E} = \frac{145.1\,\mathrm{W\,m^{-2}}}{296.8\,\mathrm{W\,m^{-2}}} = \underline{0.49}$$

4. The relation between specific humidity q and water vapour density ρ_v is: $\rho_v = \rho_a q$, where ρ_a is the density of the (moist) air. Because ρ_a is not changing significantly it can be taken out of the averaging operator:

$$\begin{aligned} Q_E &= L_v \overline{w' \rho_v'} \\ &= L_v \overline{w'(\rho_a q)'} \\ &= \rho_a L_v \overline{w'q'} \end{aligned}$$

rearrange:

$$\begin{aligned} \overline{w'q'} &= \frac{Q_E}{\rho_a L_v} \\ &= \frac{240\,\mathrm{J\,s^{-1}\,m^{-2}}}{1.188\,\mathrm{kg\,m^{-3}} \times 2.453 \times 10^6\,\mathrm{J\,kg^{-1}}} \\ &= 8.24 \times 10^{-5}\,\mathrm{m\,s^{-1}\,kg\,kg^{-1}} \\ &= \underline{0.0824\,\mathrm{m\,s^{-1}\,g\,kg^{-1}}} \end{aligned}$$

5. This covariance is already a mass flux density, because its units are mass per square meter and per second if sorted properly:

$$\begin{aligned} F_{\mathrm{CH4}} &= \overline{w' \rho_{\mathrm{CH4}}'} \\ &= 10\,\mathrm{m\,s^{-1}}\,\mu\mathrm{g\,m^{-3}} \\ &= \underline{10\,\mu\mathrm{g\,m^{-2}\,s^{-1}}} \end{aligned}$$