

Answers to Study Questions - Topic 11

1. Assume a uniform and linear warming rate of the soil:

$$\begin{aligned}\frac{\Delta T}{\Delta t} &= \frac{25.3^{\circ}\text{C} - 24.8^{\circ}\text{C}}{1 \text{ h}} \\ &= 0.5 \text{K h}^{-1} = 1.38 \times 10^{-4} \text{K s}^{-1}\end{aligned}$$

The heat flux at the surface $Q_{G(0)}$ is (Lecture 11, Slide 10):

$$\begin{aligned}Q_{G(0)} &= Q_{G(5\text{cm})} + C \frac{\Delta T}{\Delta t} \Delta z \\ &= 25 \text{ W m}^{-2} + 2 \text{ MJ m}^{-3} \text{K}^{-1} \times 1.38 \times 10^{-4} \text{K s}^{-1} \times 0.05 \text{ m} \\ &= 25 \text{ W m}^{-2} + 13.8 \text{ W m}^{-2} \\ &= \underline{38.8 \text{ W m}^{-2}}\end{aligned}$$

2. Now we have a cooling rate (i.e. negative change of temperature over time):

$$\begin{aligned}\frac{\Delta T}{\Delta t} &= \frac{7.0^{\circ}\text{C} - 7.5^{\circ}\text{C}}{1 \text{ h}} \\ &= -0.5 \text{K h}^{-1} = -1.38 \times 10^{-4} \text{K s}^{-1}\end{aligned}$$

Similar to the first example, the heat flux at the surface is:

$$\begin{aligned}Q_{G(0)} &= Q_{G(5\text{cm})} + C \frac{\Delta T}{\Delta t} \Delta z \\ &= -12 \text{ W m}^{-2} + 2 \text{ MJ m}^{-3} \text{K}^{-1} \times (-1.38 \times 10^{-4} \text{K s}^{-1}) \times 0.05 \text{ m} \\ &= -12 \text{ W m}^{-2} + (-13.88 \text{ W m}^{-2}) \\ &= \underline{-25.9 \text{ W m}^{-2}}\end{aligned}$$

3. Heat sharing refers to how the soil and the atmosphere (or two other materials) share in accepting sensible heat (i.e. Q^* - Q_E) during the

daytime and share in releasing it at night. Heat sharing is determined by the ratio of the thermal admittances of the two materials (Lecture 11, slides 27):

$$\frac{Q_H}{Q_G} = \frac{\mu_a}{\mu_s}$$

where μ_s is the thermal admittance of the soil and μ_a is the thermal admittance of the atmosphere.

4. Rearrange the heat sharing equation from Question 3:

$$Q_H = Q_G \times \frac{\mu_a}{\mu_s}$$

We can calculate μ_s by:

$$\begin{aligned}\mu_s &= \sqrt{kC} \\ &= \sqrt{0.27 \text{ W m}^{-1} \text{ K}^{-1} \times 2 \times 10^6 \text{ J m}^{-3} \text{ K}^{-1}} \\ &= 735 \text{ J m}^{-2} \text{ K}^{-1} \text{ s}^{-1/2}\end{aligned}$$

Hence:

$$\begin{aligned}Q_H &= Q_G \times \frac{\mu_a}{\mu_s} \\ &= 38.8 \text{ W m}^{-2} \times \frac{5000 \text{ J m}^{-2} \text{ K}^{-1} \text{ s}^{-1/2}}{735 \text{ J m}^{-2} \text{ K}^{-1} \text{ s}^{-1/2}} \\ &= \underline{264 \text{ W m}^{-2}}\end{aligned}$$