Photo: A. Christen

20 Velocity profile laws.

Today's learning objectives

- Describe how the mean velocity profile is linked to the momentum flux.
- Understand the 'famous' logarithmic wind profile equation.

Photo: A. Christen

The vertical profile of horizontal wind

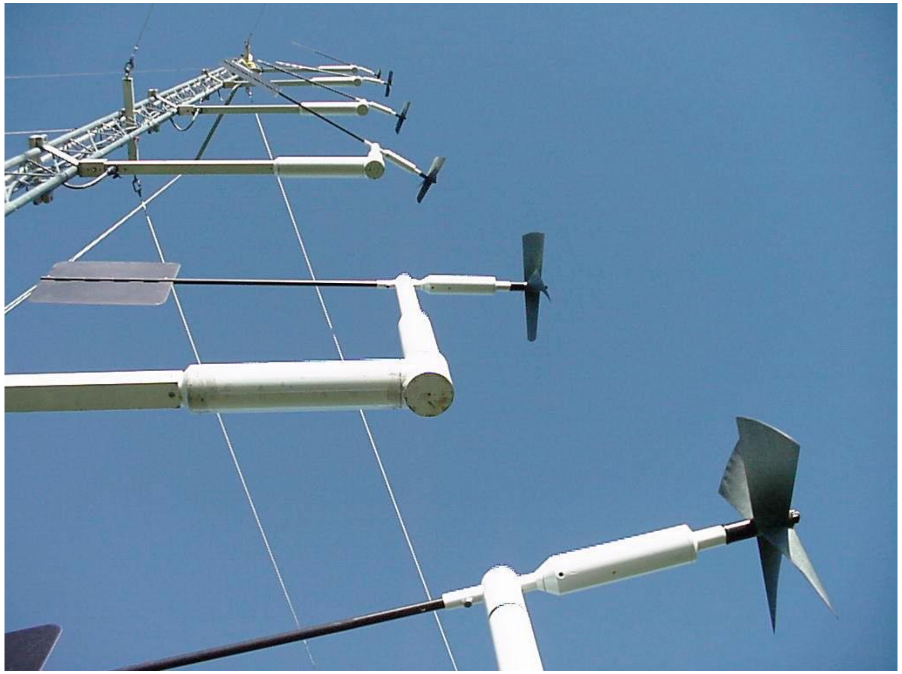
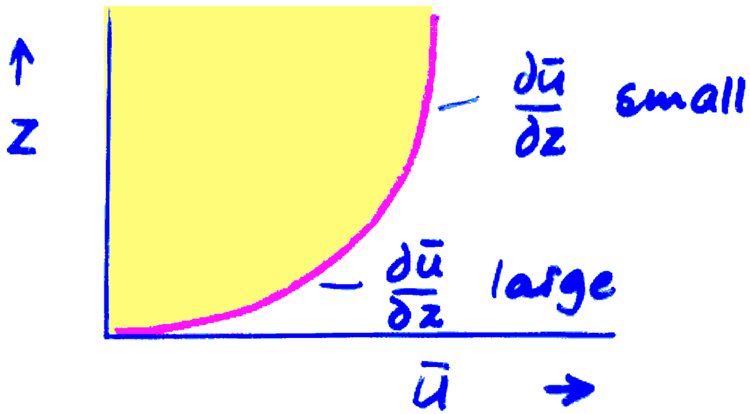


Photo: A. Christen

Series of anemometers at different heights z over an extensive flat surface gives:



This is the vertical profile of mean horizontal wind $\bar{u}$ and shows that the **wind gradient** (or 'wind shear') decreases as z increases.

The mixing length

Assume turbulent whirl at level (z+l) with mean velocity $\bar{u}_{(z+l)}$ moves down to z where mean velocity $\bar{u}(z)$ is less, by u':

$$u' = \overline{u}_{(z+\ell)} - \overline{u}_{(z)}$$

so

$$u' = \ell \frac{\partial u}{\partial z}$$

i.e. extra velocity = increment in height x rate of change of velocity with height.

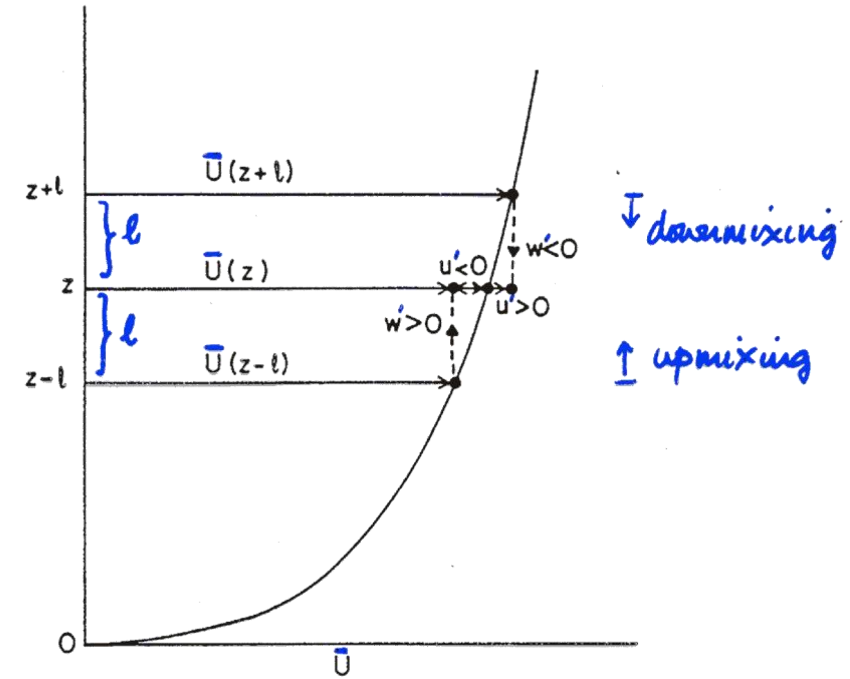


Fig. 9.1 Schematic of mean velocity profile in the lower atmosphere and expected correlations between the longitudinal and vertical velocity fluctuations due to fluid parcels coming from above or below before mixing with the surrounding fluid.

The characteristic height for mixing to occur is the **mixing length l** and is likely related to the mean size of eddies.

The mixing length approach

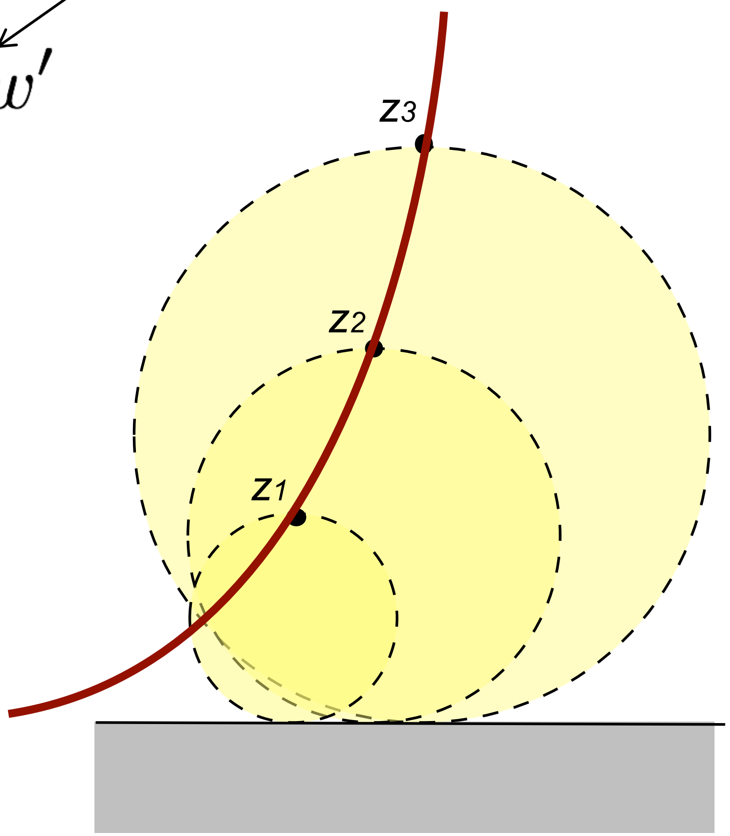
Tau = Reynolds stress or momentum flux

If turbulence is assumed to be isotropic (eddies are symmetric) $u' \approx w'$ and $\tau = -\rho\, u'w'$ and $\tau \approx \rho u_*^2$ (see Topic 19) it follows $u' \approx w' \approx u_*$ and we can write

Friction velocity (m s^{-1})

$$\frac{\partial \overline{u}}{\partial z} = \frac{u_*}{\ell} \qquad (1)$$

i.e. the **wind gradient is inversely related to the size of the eddies**. As we approach the boundary (ground) the spectrum of eddy sizes is restricted by the physical barrier.



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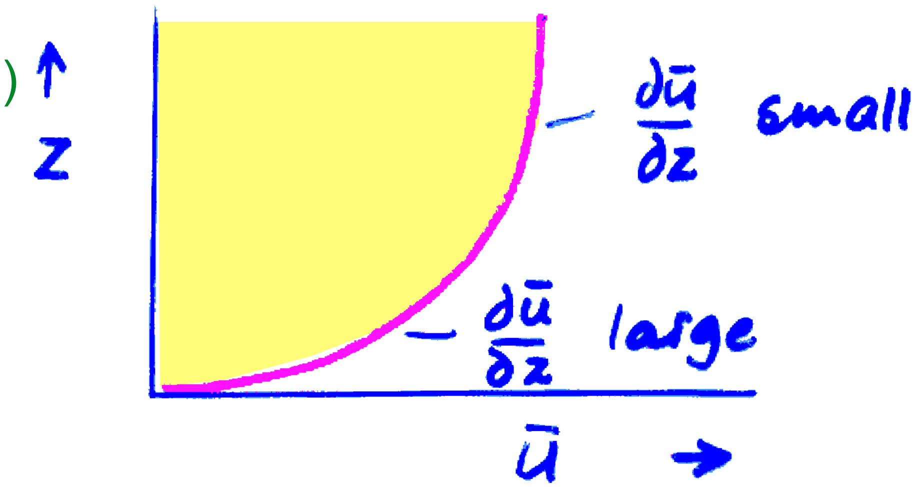

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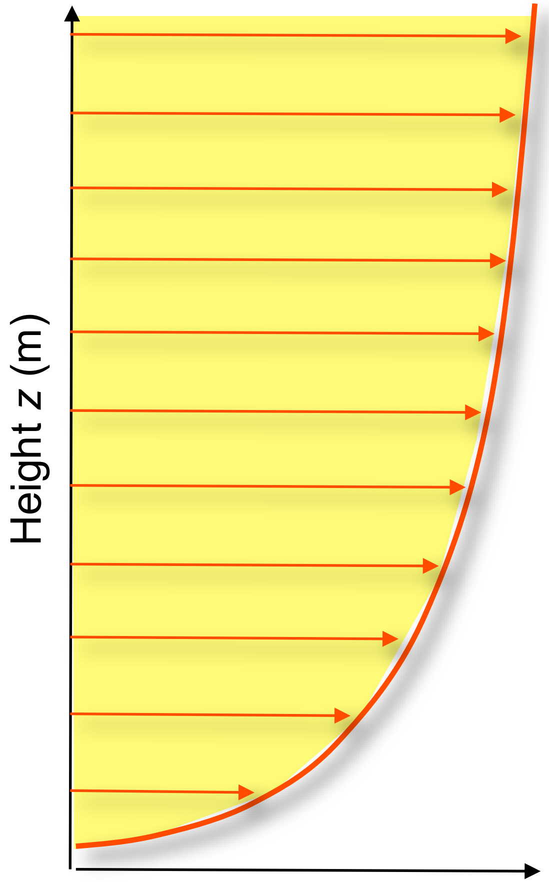
The vertical profile of horizontal wind



The form of the vertical profile of horizontal wind is found to be

$$\frac{\partial \overline{u}}{\partial z} = a(z)^{-b} \qquad (2)$$

The form says $\bar{u}$ is proportional to the **logarithm of z**, which was extensively verified in the field and laboratory.

von Kármán's constant

So if the eddy size is proportional to the distance to the ground, let's set

$$\ell = k(z)^b$$

von Kármán's constant

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$$\ell = k(z)^b$$

Substitution of ℓ into (1) gives

$$\frac{\partial \overline{u}}{\partial z} = \frac{u_*}{k}(z)^{-b}$$

i.e. in (2)

$$a = u_*/k$$

In neutral stability b = 1 so

$$\ell = kz$$

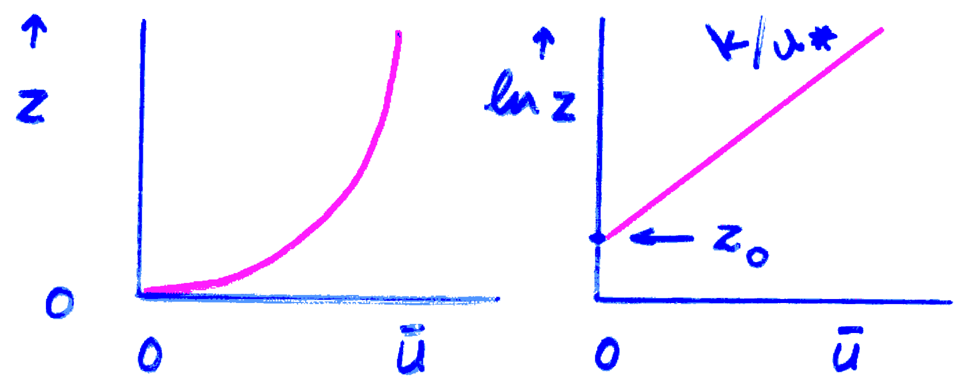
and therefore

$$\frac{\partial \overline{u}}{\partial z} = \frac{u_*}{kz} \qquad (3)$$

The constant k is **von Kármán's constant** (0.41)

The logarithmic wind law

Integration of (3) requires knowledge of the exact height where is $\bar{u}$ zero (integration constant). In practice this depends on the roughness of the ground so we set a roughness length (z_0, units m) and restrict the equation to $z > z_0$.



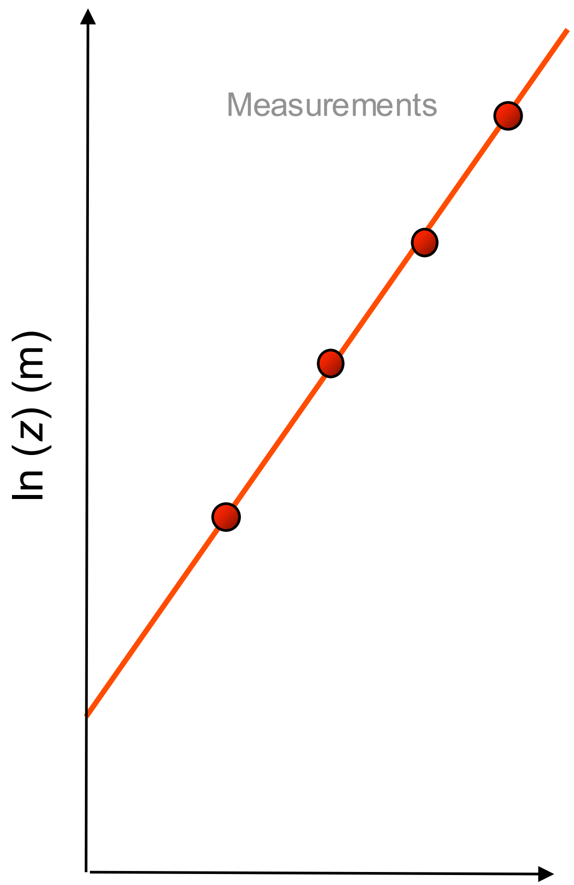
$$\overline{u}_{(z)} = \frac{u_*}{k} \ln\left(\frac{z}{z_0}\right)$$ ★

This is the 'famous' log wind law for neutral conditions.

The general form for measurements in two levels:

$$\overline{u}_2 - \overline{u}_1 = \frac{u_*}{k} \ln\left(\frac{z_2}{z_1}\right)$$ ★

Linear-log plot of the neutral wind profile



Horizontal mean wind $\bar{u}$ (m s^{-1})

In neutral conditions slope in this linear-log graph is exactly linear and both, u_* and z_0 can be determined from graphical analysis of wind data from several levels.

$$\overline{u}_{(z)} = \frac{u_*}{k} \ln\left(\frac{z}{z_0}\right) \quad \star$$

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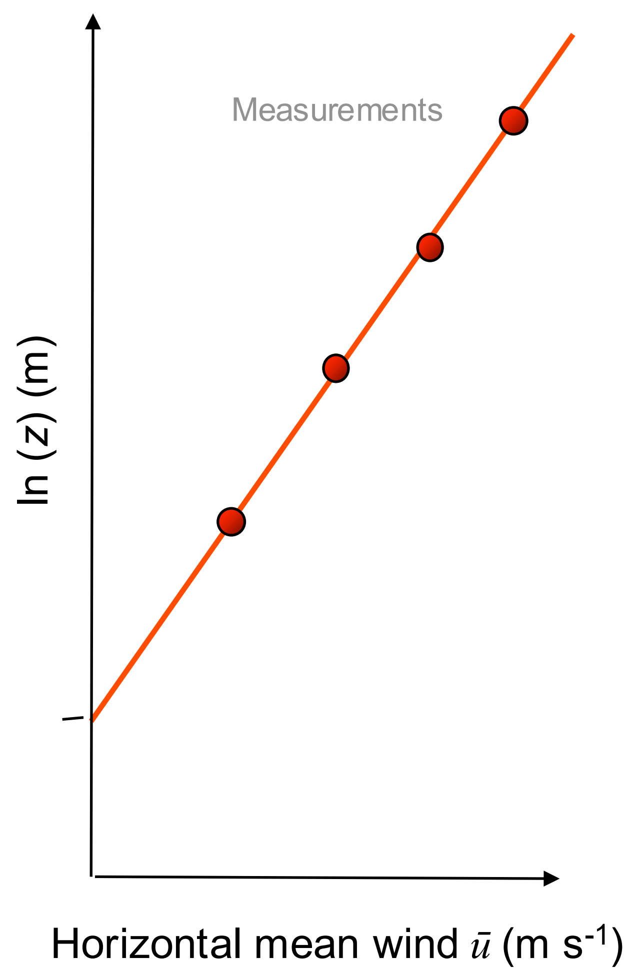
What is the intercept?



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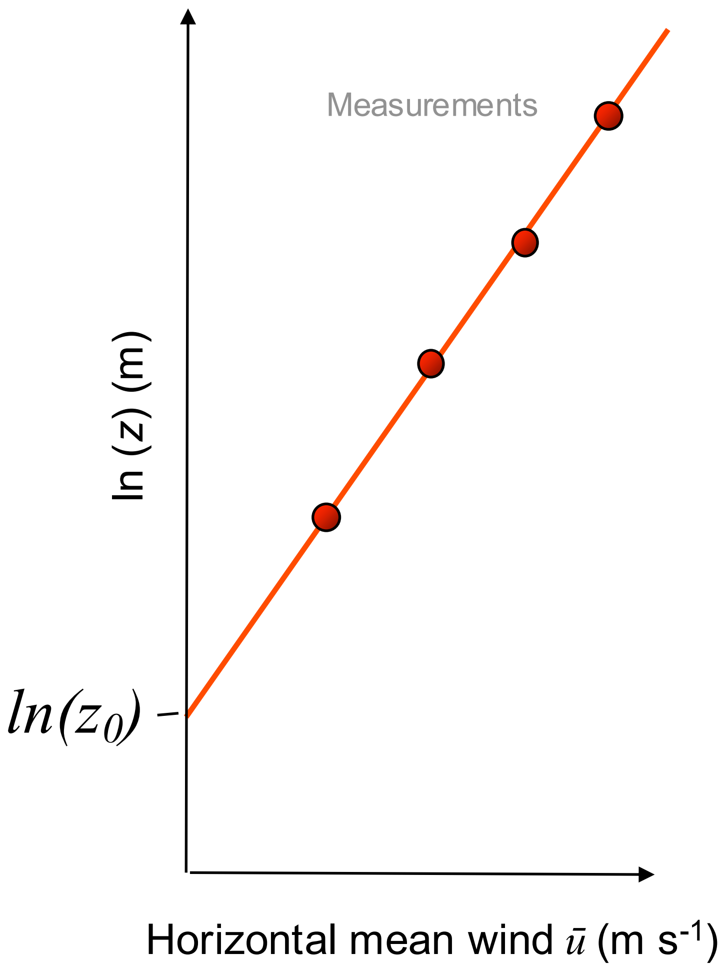

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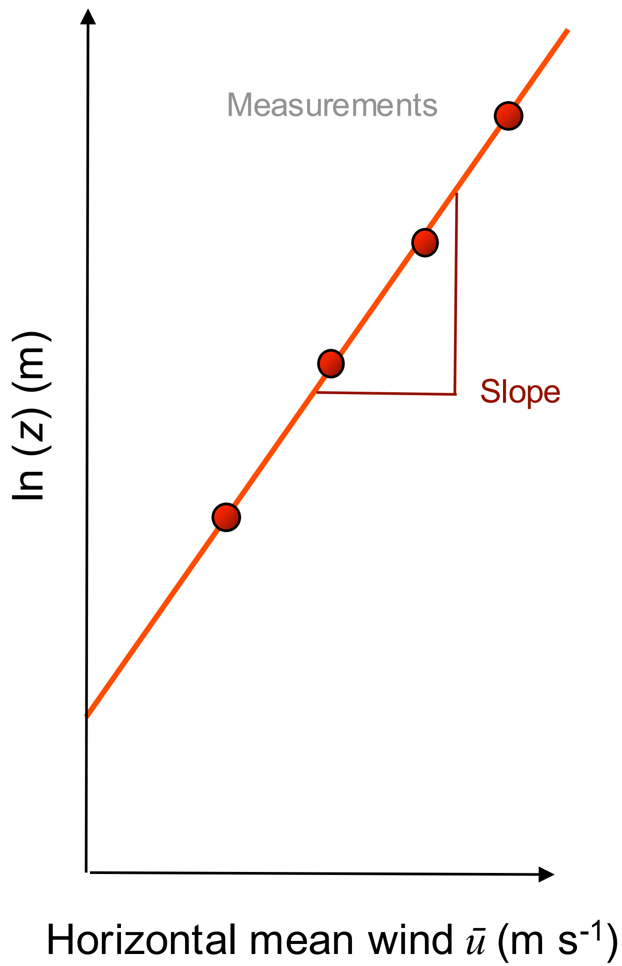
What is the slope?



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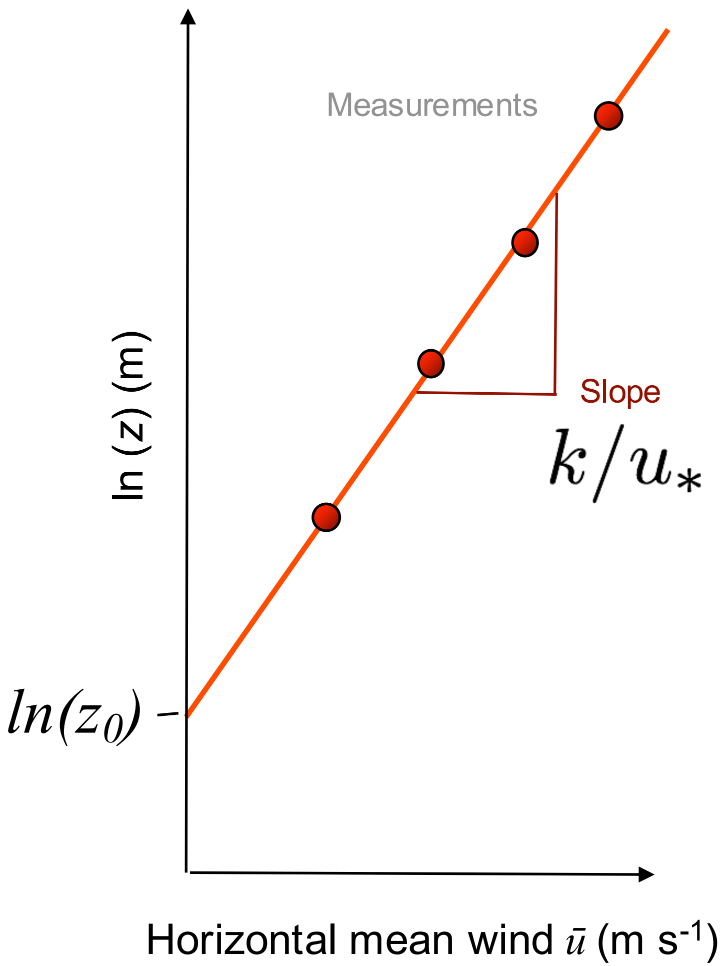

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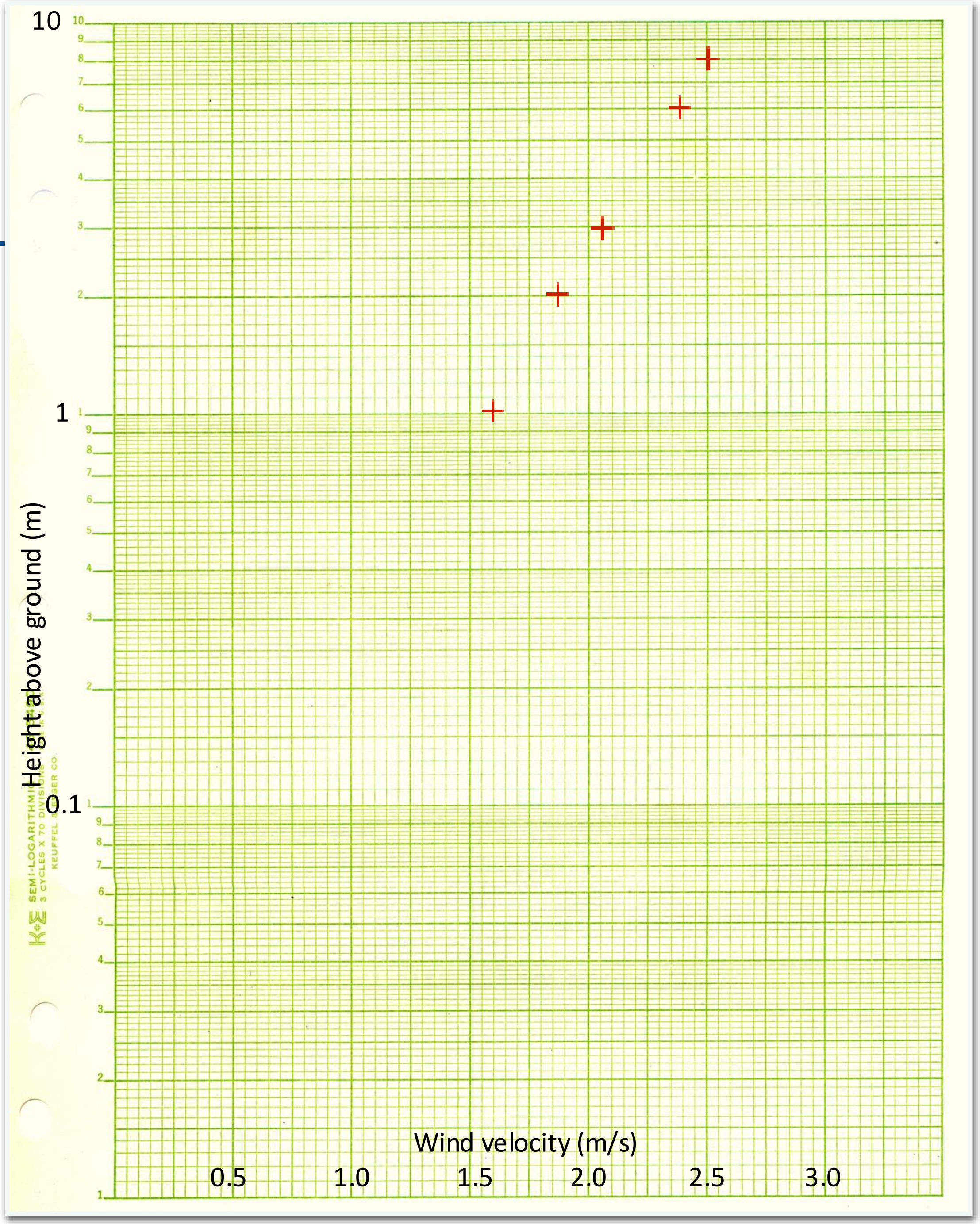
$$\overline{u}_{(z)} = \frac{u_*}{k} \ln\left(\frac{z}{z_0}\right) \quad \star$$

Semi-log paper

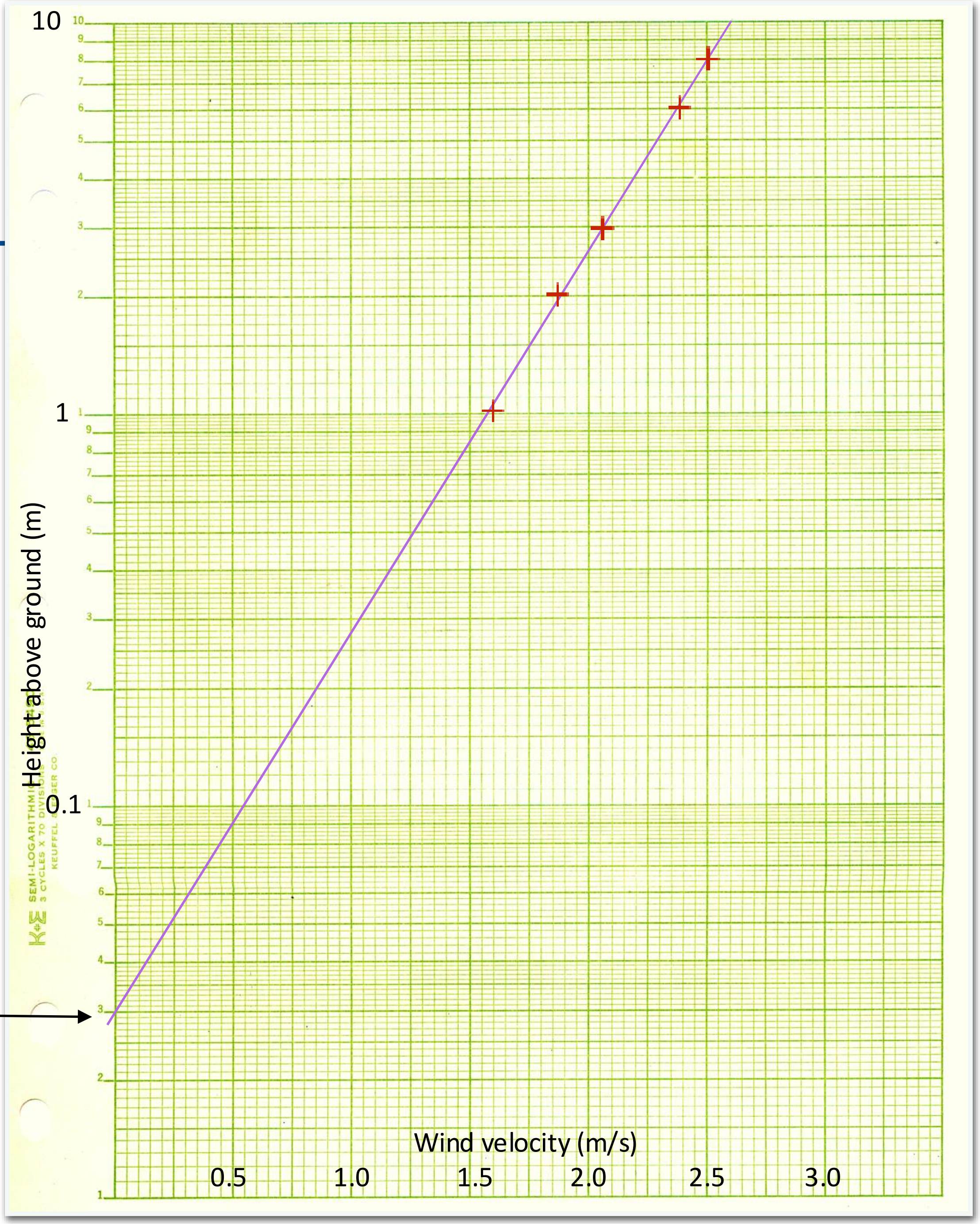
On this paper, you can directly enter wind velocity on the x-axis (linear scale) and height on the y-axis (logarithmic scale).

First label your own axes (orders of magnitude on y-axis) to fit data optimally on paper. Then enter data points.



Semi-log paper

You can **fit a line** (e.g. by hand using a ruler). The intercept with the y-axis gives you an estimate of z_0.

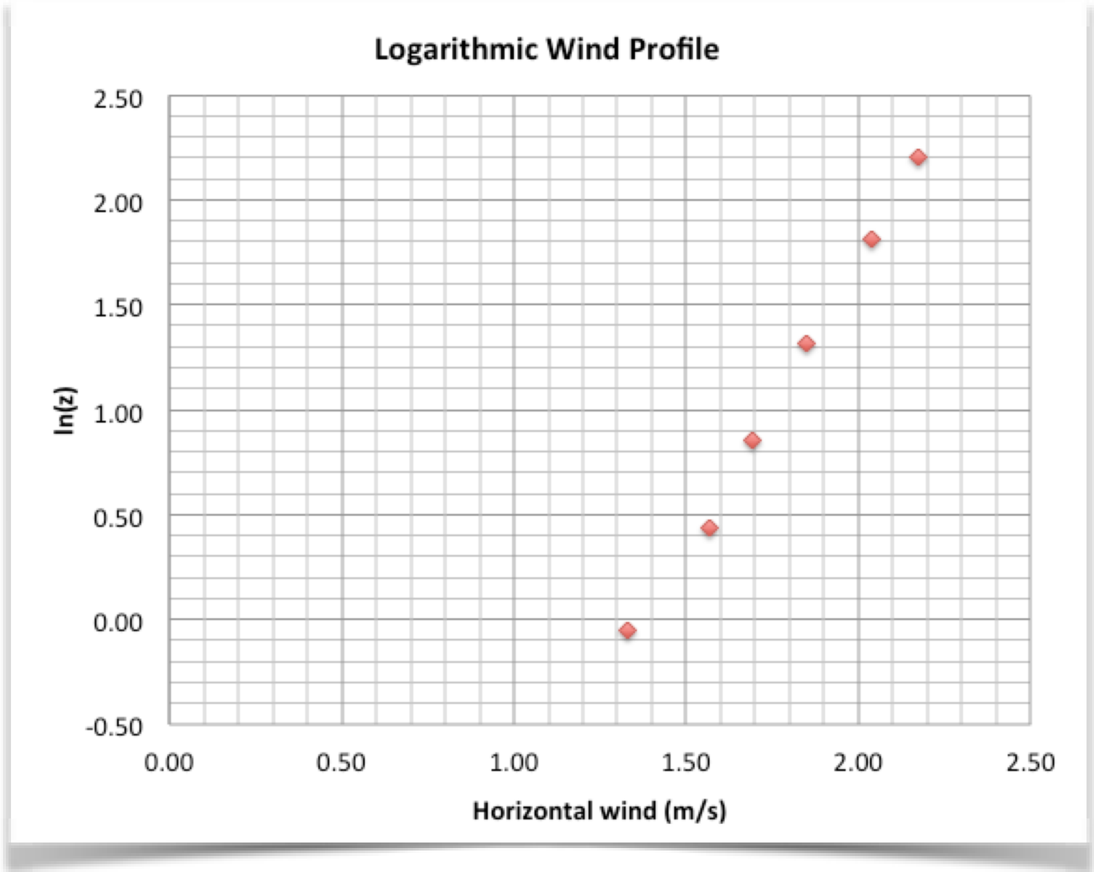


$z_0 = 0.03$ m

Example to graph the profile in a spreadsheet software

Alternatively can use spreadsheet and convert z to $\ln(z)$ to graph $\ln(z)$ vs. wind (u), then plot as a scatter plot

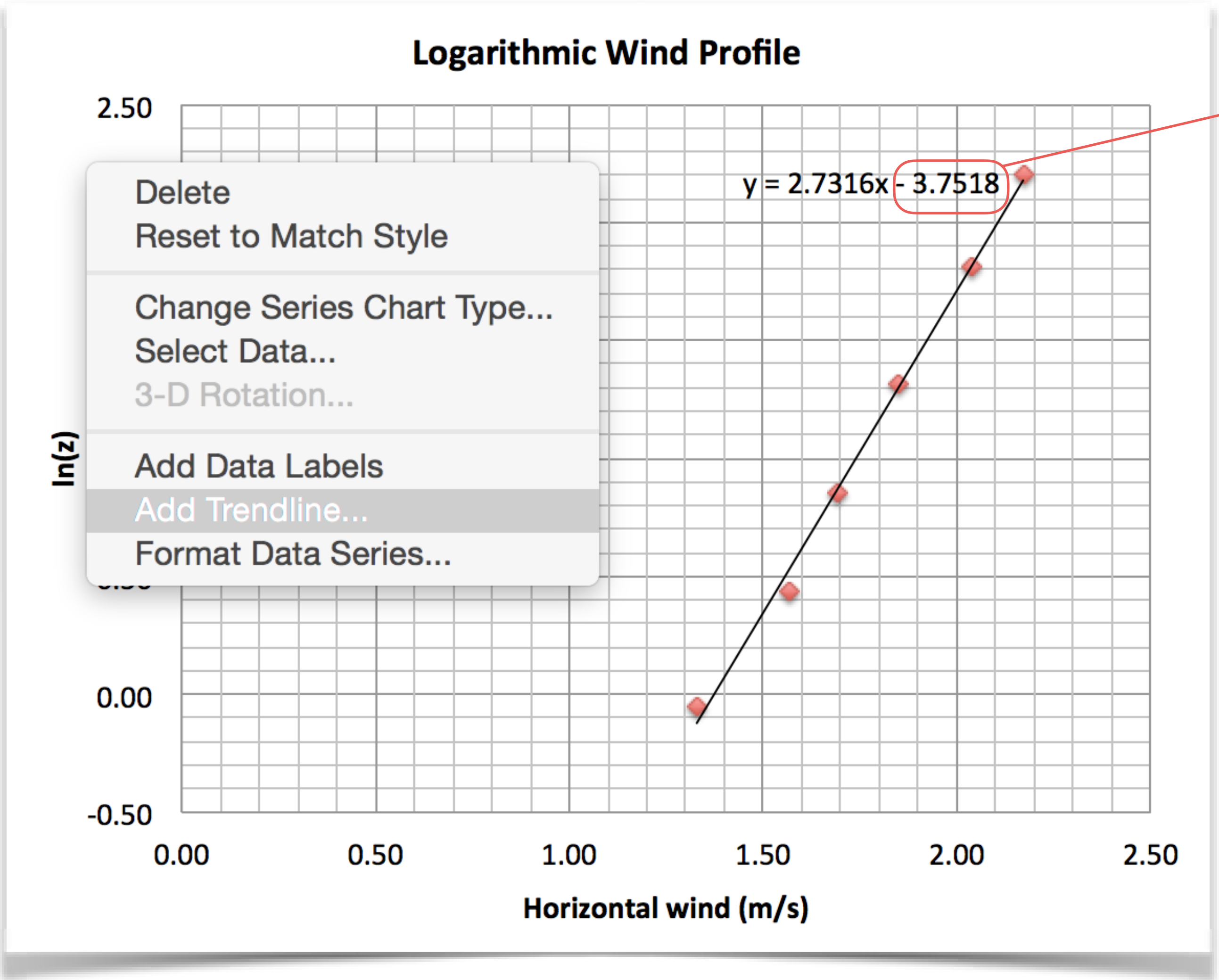
	A	B	C
1	z (m)	ln (z)	u (m/s)
2	0.95	=LN(A2)	1.33
3	1.55	0.44	1.57
4	2.35	0.85	1.69
5	3.72	1.31	1.85
6	6.15	1.82	2.04
7	9.05	2.20	2.17



X values: =Sheet1!$C$2:$C$6

Y values: =Sheet1!$B$2:$B$6

Example to determine z_0 in the spreadsheet software



$\ln(z_0) = \text{intercept}$

hence:

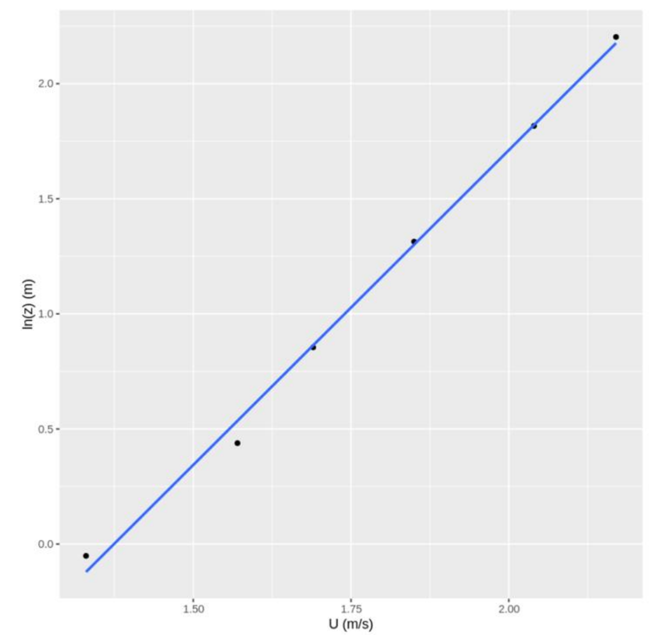
$z_0 = \exp(\text{intercept})$

In most software, you can create a linear fit through a set of points. The y-intercept (offset) of the linear fit is equal to $\ln(z_0)$.

Example from R

```r
# Load example data
data <- read.csv(file = 'Lecture 20 Log wind profile example.csv', header = TRUE)

# Create new column for ln(z)
data$lnz <- log(data$z)

# Display data
head(data)

# Plot wind speed vs. height
suppressMessages(library("ggplot2")) # load plotting package first

ggplot(data, aes(x = U, y = lnz)) +
  geom_point() +
  labs(
    x = "U (m/s)",
    y = "ln(z) (m)") +
  geom_smooth(method = "lm", se = FALSE)


# Fit line through data. Note for this use the function lm
modelFit <- lm(lnz~U, data=data)

# estimate z0.
z0 <- exp(summary(modelFit)$coefficients[1])

cat("z0 is: ", round(z0,3), "m", "\n")

# To estimate the slope, use:
slope <- summary(modelFit)$coefficients[2]
```

	z	U	lnz
	<dbl>	<dbl>	<dbl>
1	0.95	1.33	-0.05129329
2	1.55	1.57	0.43825493
3	2.35	1.69	0.85441533
4	3.72	1.85	1.31372367
5	6.15	2.04	1.81645208
6	9.05	2.17	2.20276476



Test your knowledge - using Excel or R

Calculate z_0 based on the data provided in the in-class example

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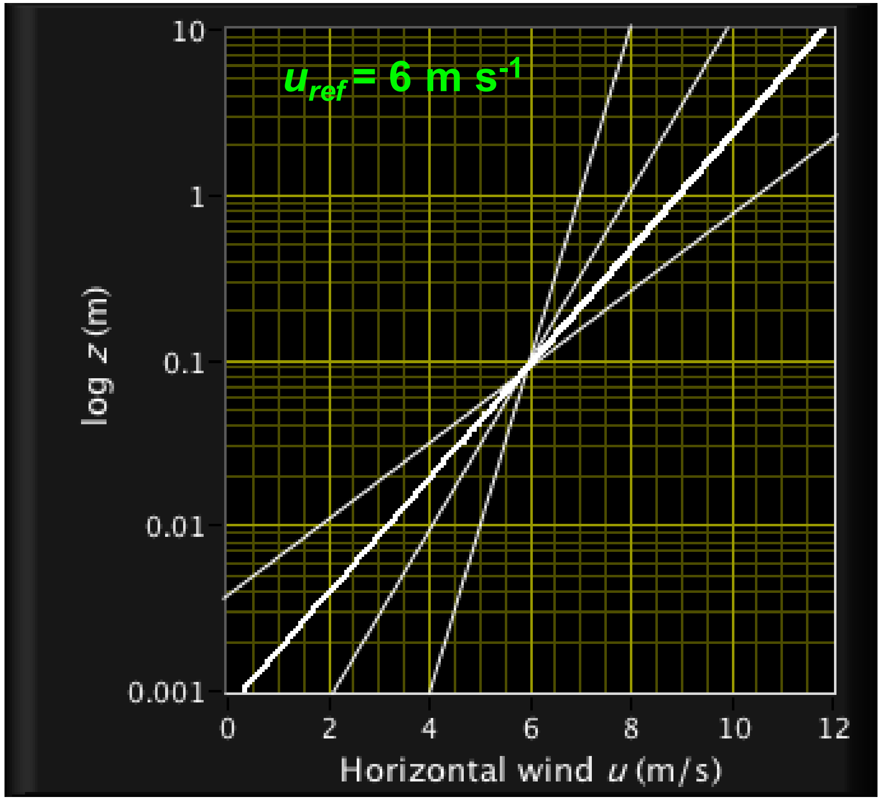
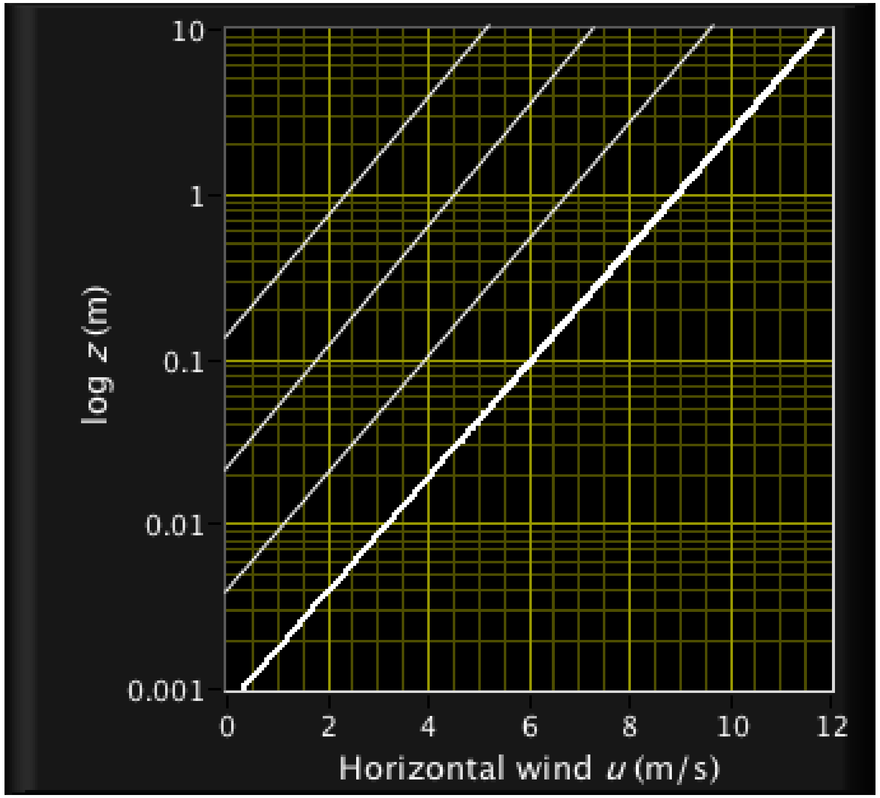
Relation between z_0 and mean wind speed

Same reference velocity u_{ref}
but different z_0 and shear stress



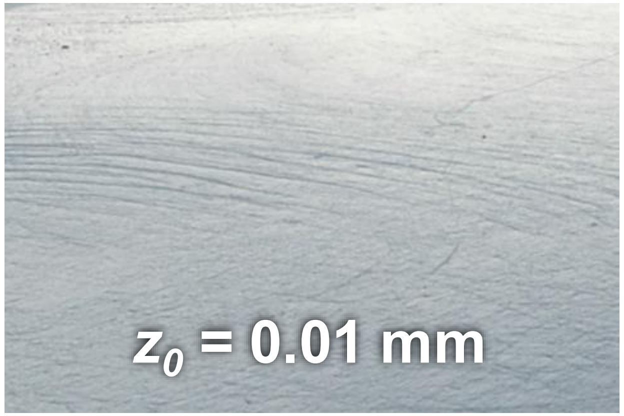
Same shear stress
but different roughness



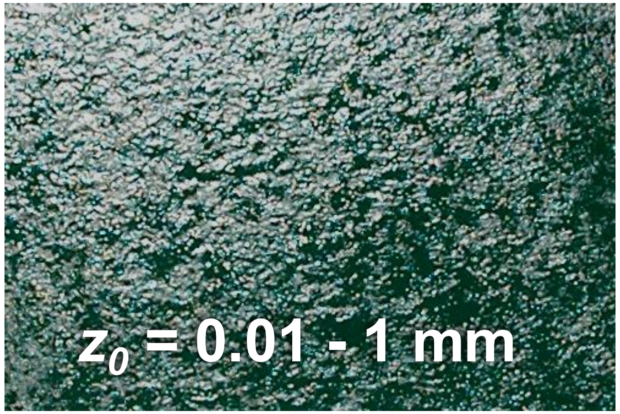
See course webpage for interactive version: https://geog321.github.io/applets/neutralwind/

Roughness length z_0 - examples

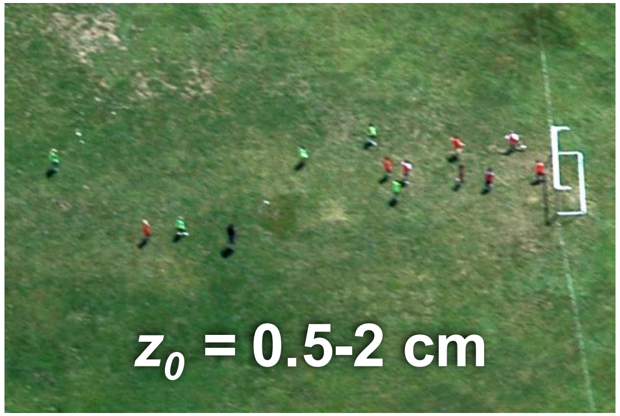
Ice



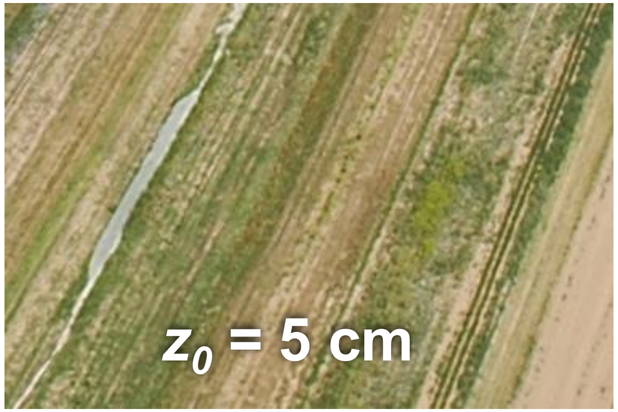
Water



Short grass

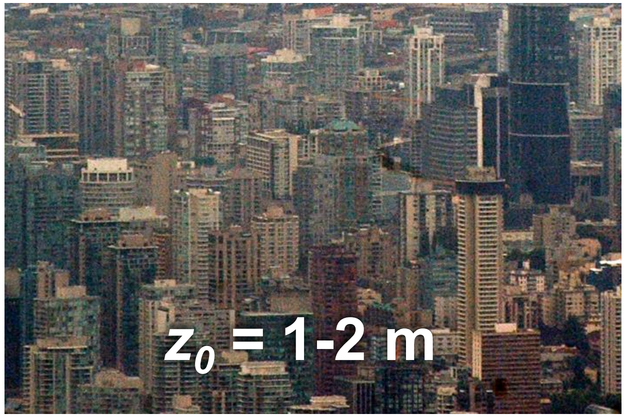


Crops



Forest

Urban

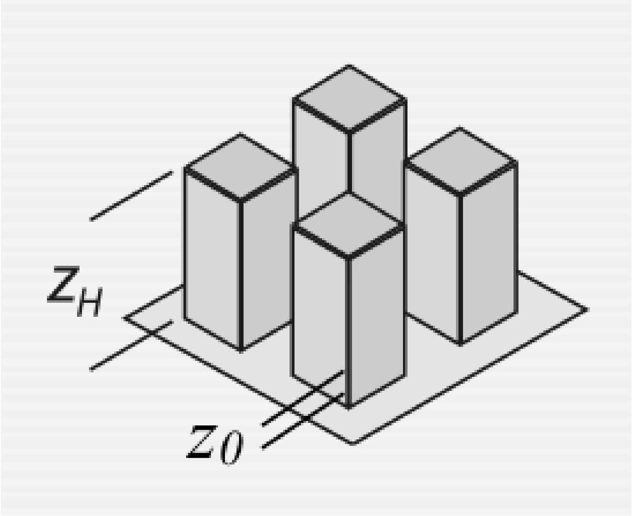


Photos: A. Christen / R. Vogt

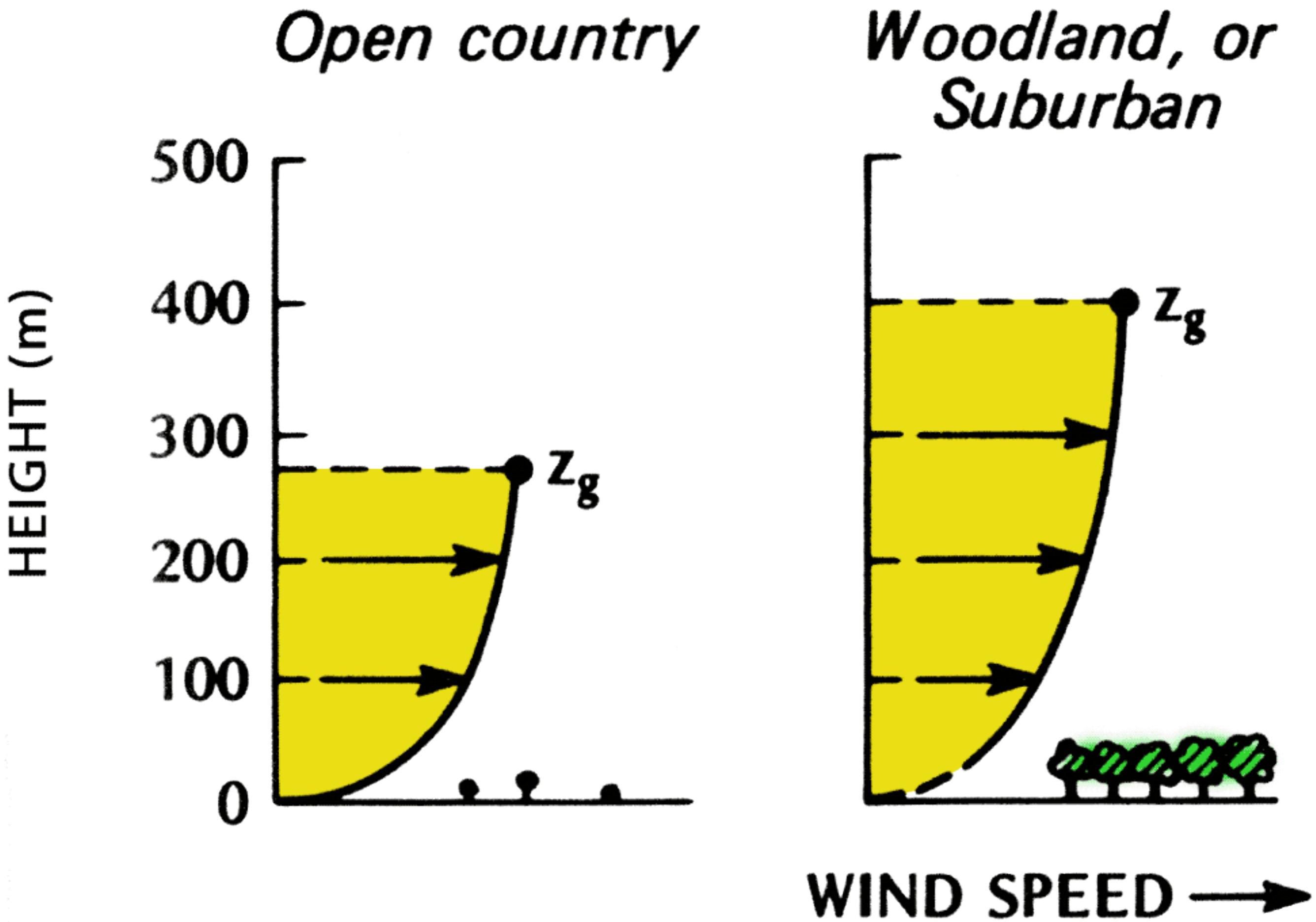
Estimating the roughness length z_0

If observations are not available, a first order estimate of z_0 can be obtained from a geometric analysis of the surface roughness elements. A traditional rule-of-thumb gives

$$z_0 \approx 0.1 z_h \quad \star$$



Wind profiles over different surfaces

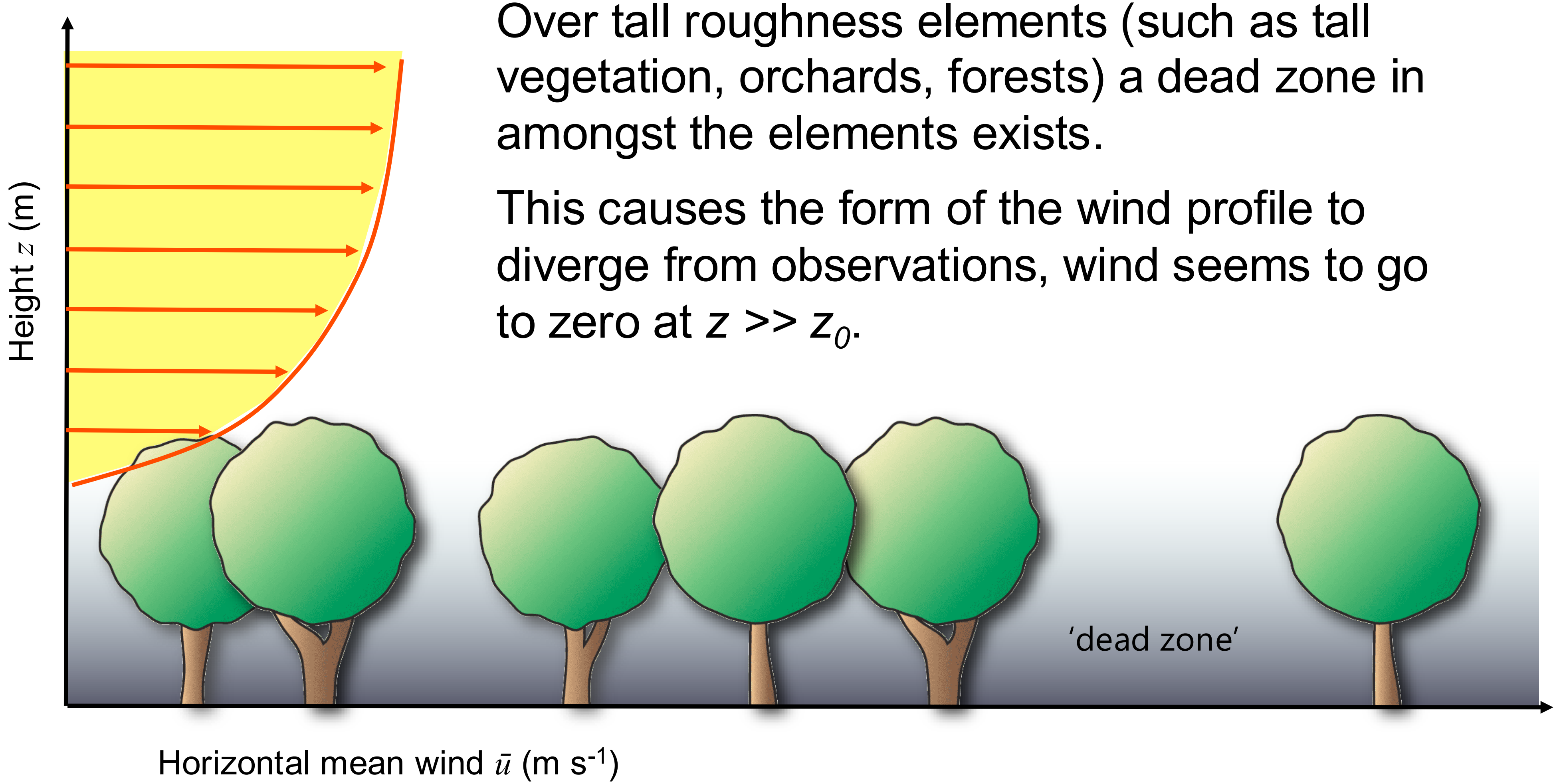


T.R. Oke (1987): 'Boundary Layer Climates' 2nd Edition.

Tall roughness

Over tall roughness elements (such as tall vegetation, orchards, forests) a dead zone in amongst the elements exists.

This causes the form of the wind profile to diverge from observations, wind seems to go to zero at $z >> z_0$.

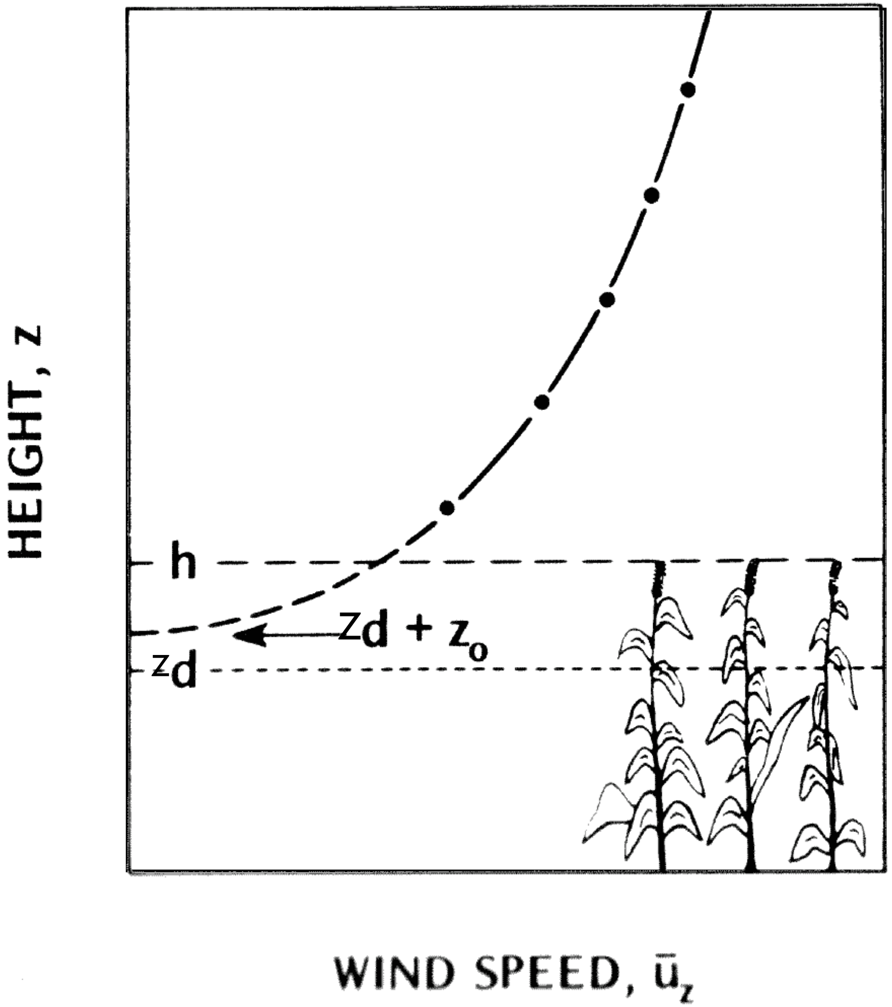


Zero-plane displacement z_d

We introduce an effective height of mean drag, the zero-plane displacement z_d:

$$\overline{u}_{(z)} = \frac{u_*}{k} \ln\left(\frac{z - z_d}{z_0}\right) \star$$

Over tall roughness elements the straight line plot of $\bar{u}$ vs. $\ln(z)$ breaks down - so we need to introduce an effective height of mean drag - zero plane displacement z_d.



T.R. Oke (1987): 'Boundary Layer Climates' 2nd Edition.

Estimation of zero-plane displacement z_d

It is possible to determine z_d from analysis of wind observations (iteration using trial values to find z_d which straightens plot $\bar{u}$ vs. $\ln(z - z_d)$ under neutral conditions).

A rule-of-thumb says:

$$z_d \approx \frac{2}{3} z_h$$

but it omits density consideration.

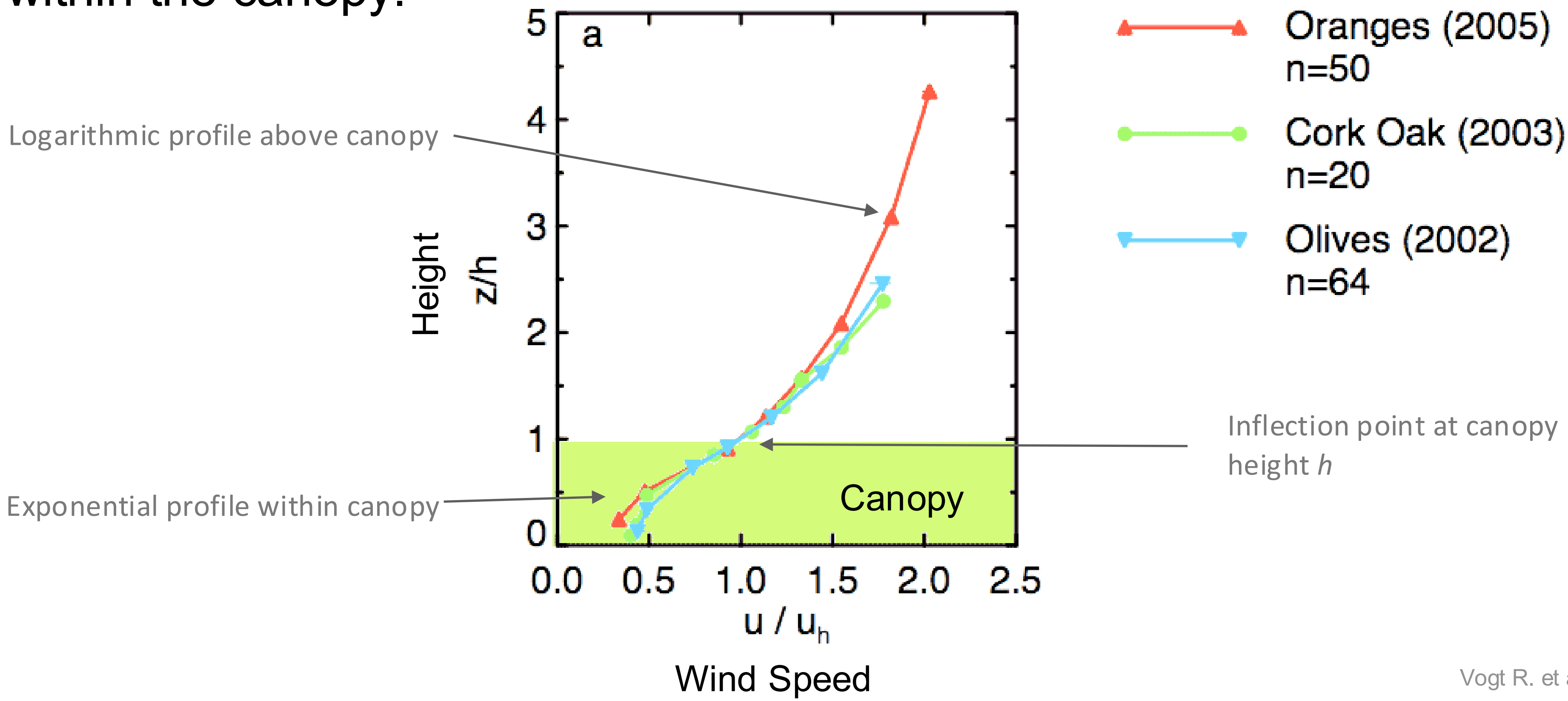
$\bar{u}_z$ should reach zero at the height $z_d + z_0$.

However, this is not observed in vegetation canopies.

We observe wind speed > 0 even below $z_d + z_0$.

The vertical profile of horizontal wind.

In most canopies (forests, orchards, crops) we find the logarithmic wind profile well above the mean canopy height *h*, and an exponential profile within the canopy.



Vogt R. et al. (2006)

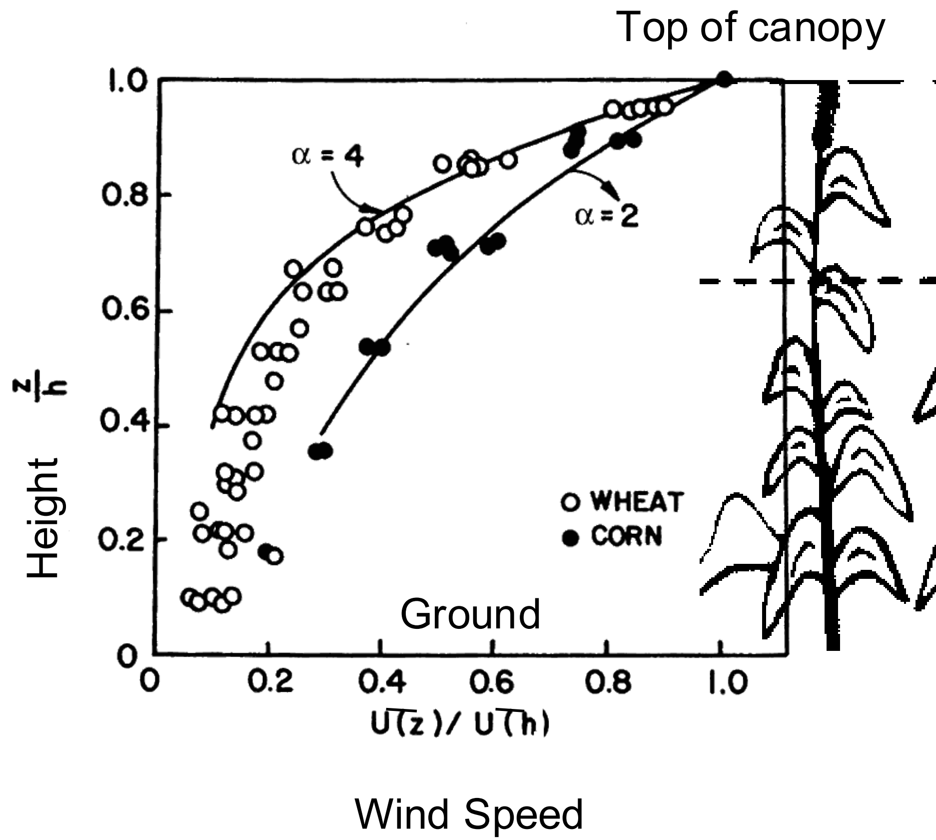
The exponential profile within plant canopies

The mean wind profile inside homogeneous canopies can be approximated as exponential function ($z < h$):

$$\overline{u}_z = \overline{u}_h \, e^{\left[\alpha\left(\frac{z}{h}-1\right)\right]}$$

α is an empirical constant, the canopy's wind extinction coefficient.

Wind extinction coefficients	α
Wheat	4
Corn	2
Sunflower	1.3
Spruce	2.4



S. P. Arya (1998): 'Introduction to Micrmeteorology'

Take home points

- The wind profile in the surface layer follows the **logarithmic law**.
- The integration constant of the log-law is the **roughness length z_0**, which depends on the surface's roughness.
- Over tall roughness elements the straight line plot of $\bar{u}$ vs. $\ln(z)$ breaks down - so we need to introduce an effective height of mean drag - **zero plane displacement z_d**.