High Arctic Landscape on Fosheim Peninsula, Ellesmere Island - a rapidly warming Arctic causes permafrost thawing and mass movements (Photo: A. Cassidy, UBC Geography Graduate Student)

12 Modelling sub-surface temperatures

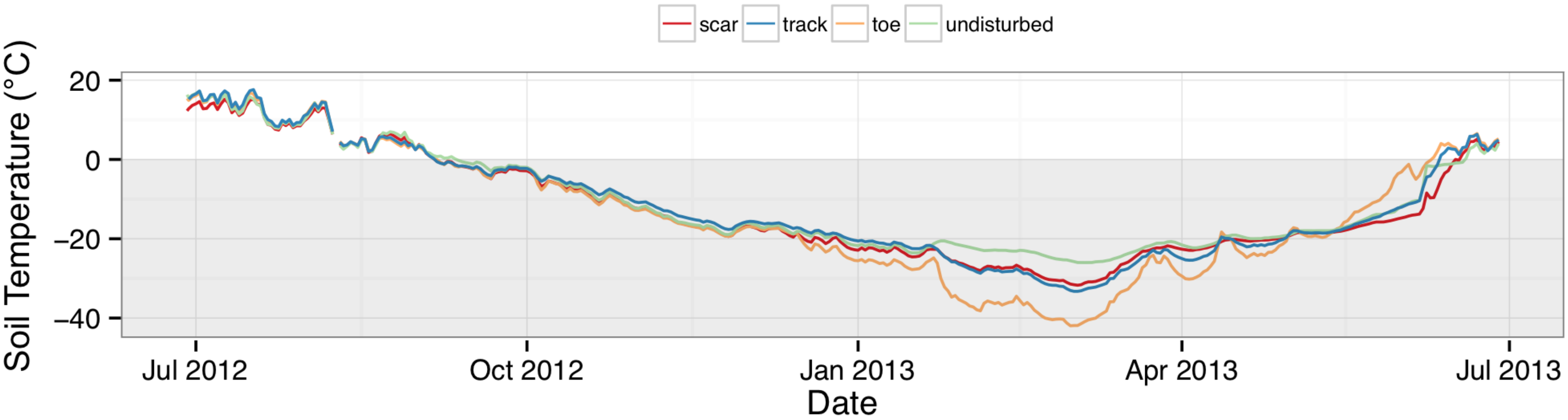
Learning objectives

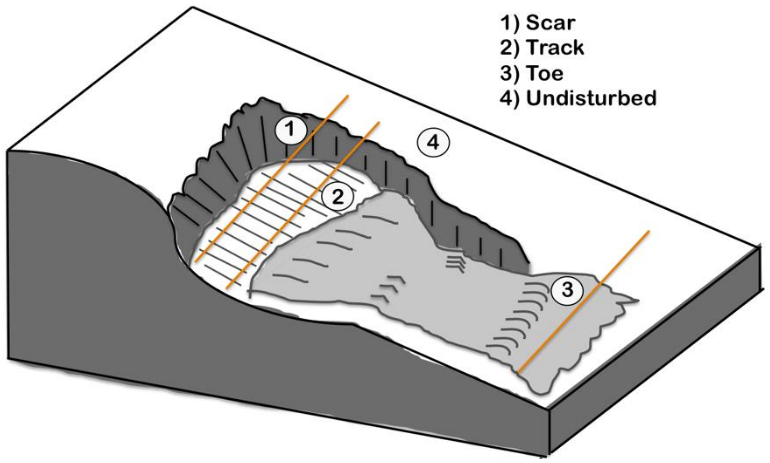
- Describe how can we predict soil temperature using a selected solution to the heat conduction equation.
- Explain how soil temperature waves move in soils.

Photo: A. Cassidy, UBC Geography


Photo: A. Cassidy, UBC Geography

Year-long near-surface soil temperatures





Arctic Bogs Hold Another Global Warming Risk That Could Spiral Out of Control

As warming brings earlier spring rains in the Arctic, more permafrost thaws, releasing more methane in a difficult-to-stop feedback loop, research shows.

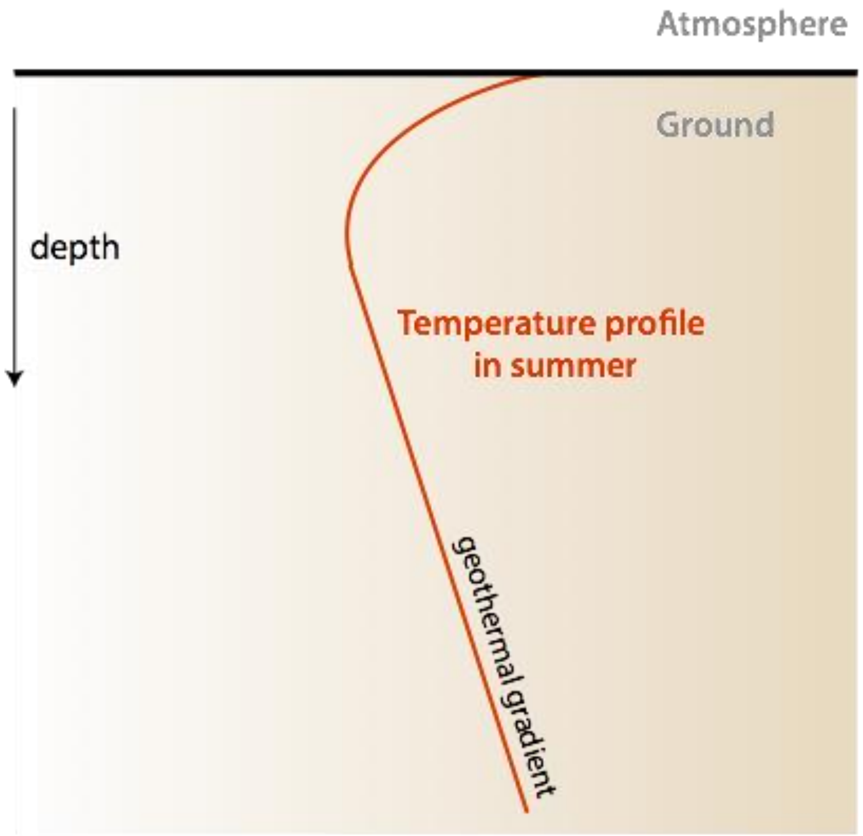
BY PHIL MCKENNA Follow @mckennapr

FEB 19, 2019

A doubling of the rate of methane released in the Arctic could have consequences that climate change projections don't currently take into account. Credit: S Hillebrand/USFWS

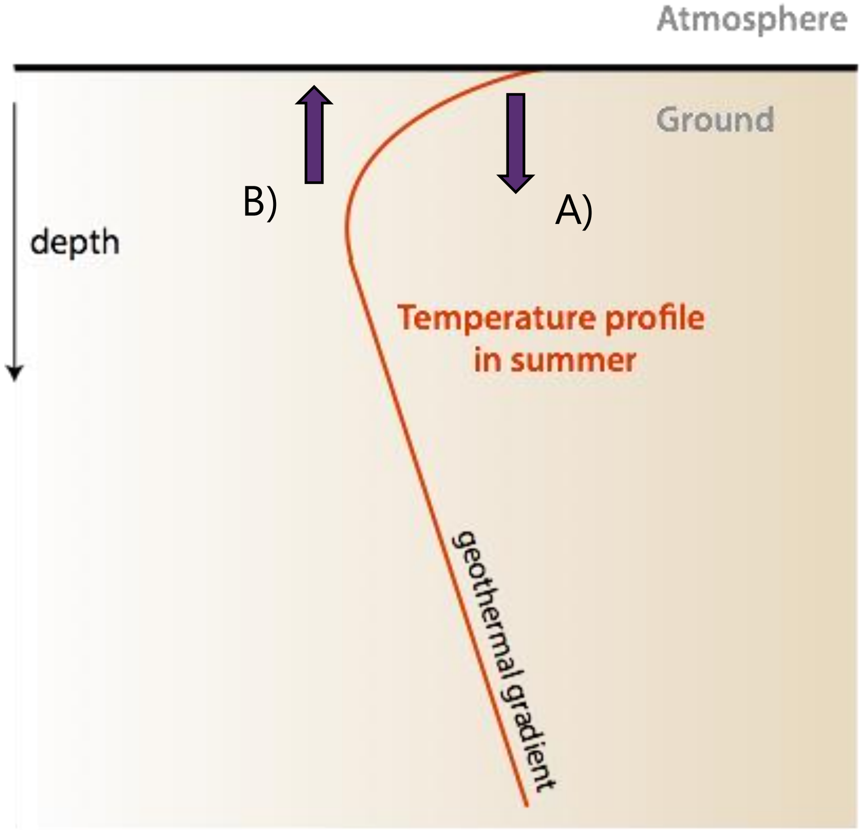
Source: https://insideclimatenews.org/news/19022019/arctic-bogs-permafrost-thaw-methane-climate-change-feedback-loop

Vertical temperature profile in ground

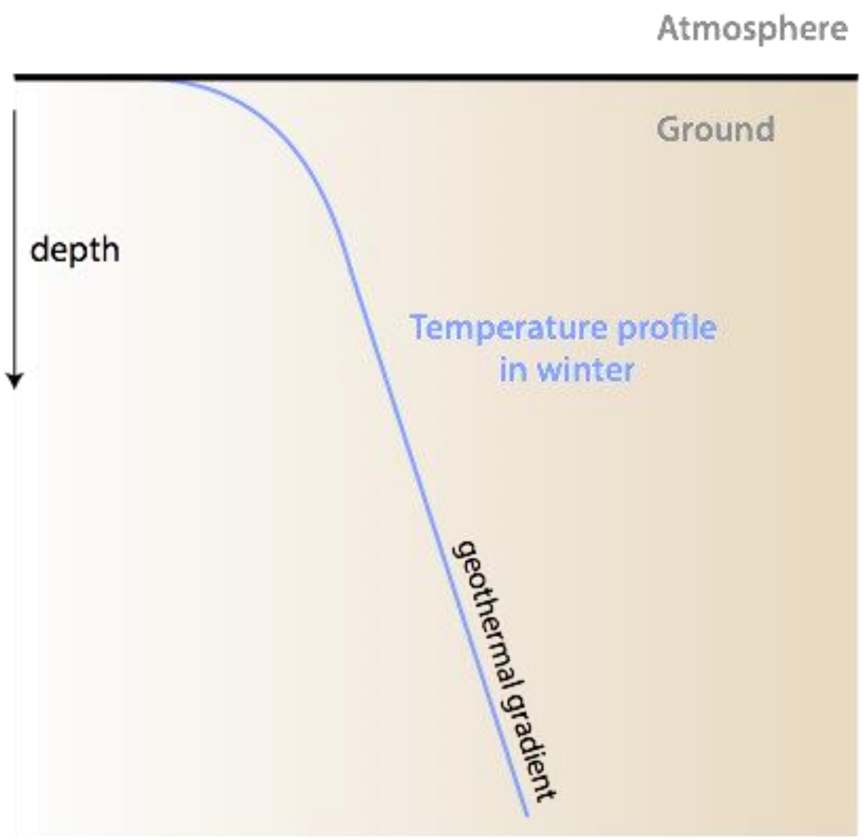


What direction is the soil heat flux (Q_G) at the surface?

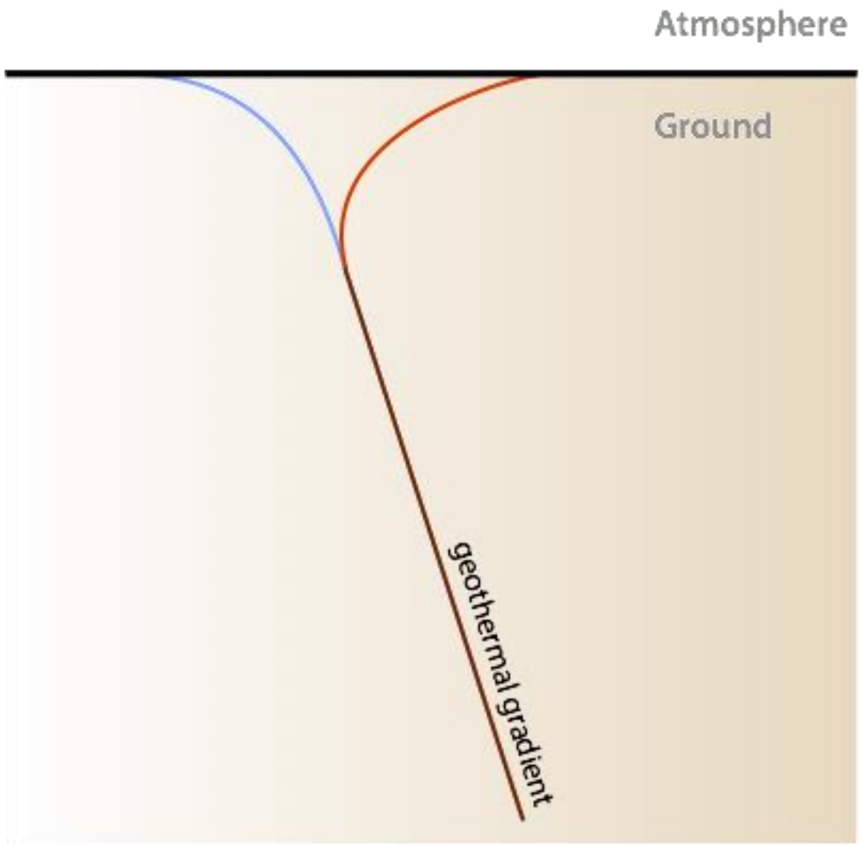
A) Downward (positive)
B) Upward (negative)



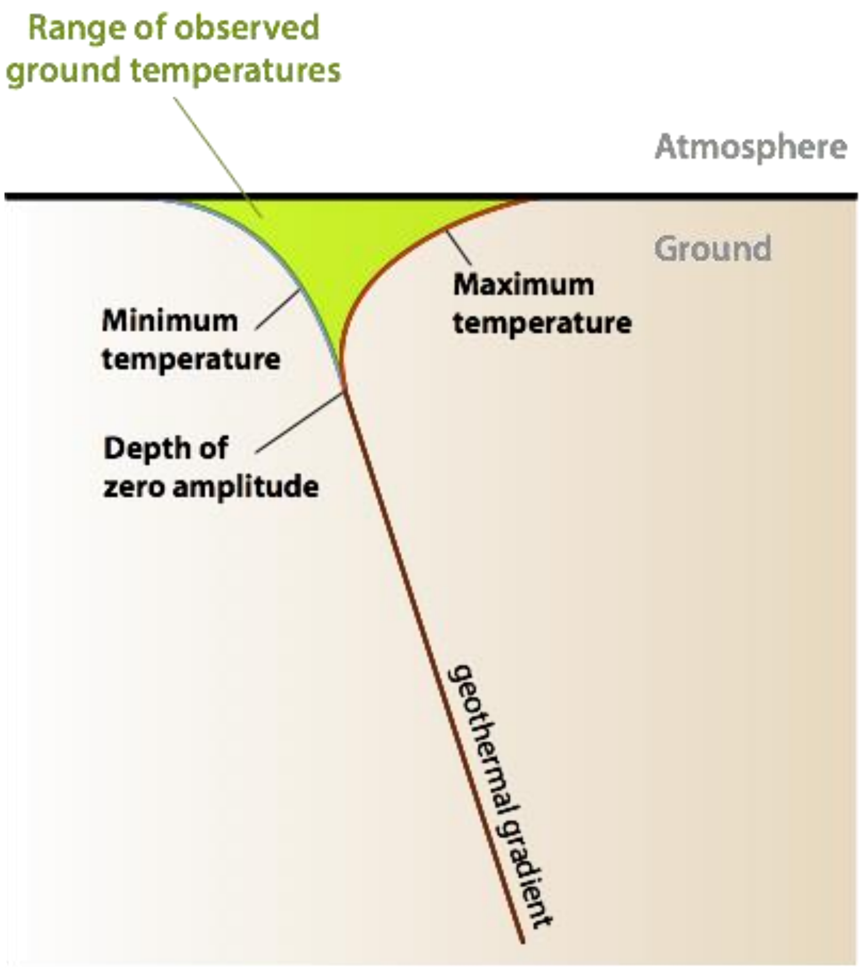
Vertical temperature profile in ground



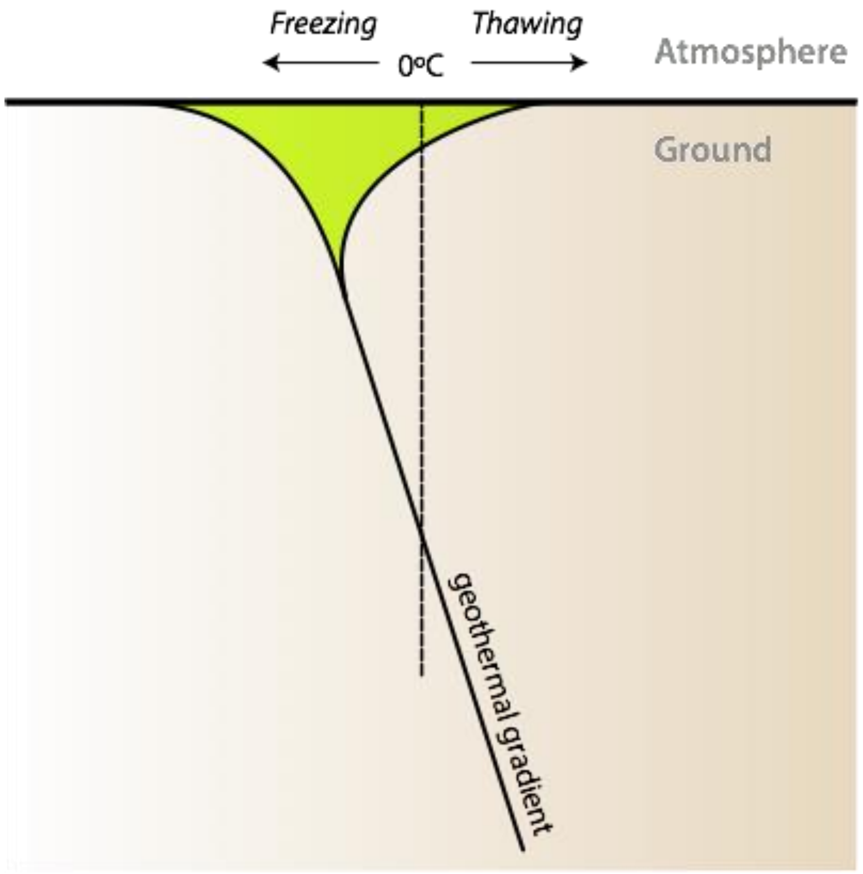
Vertical temperature profile in ground



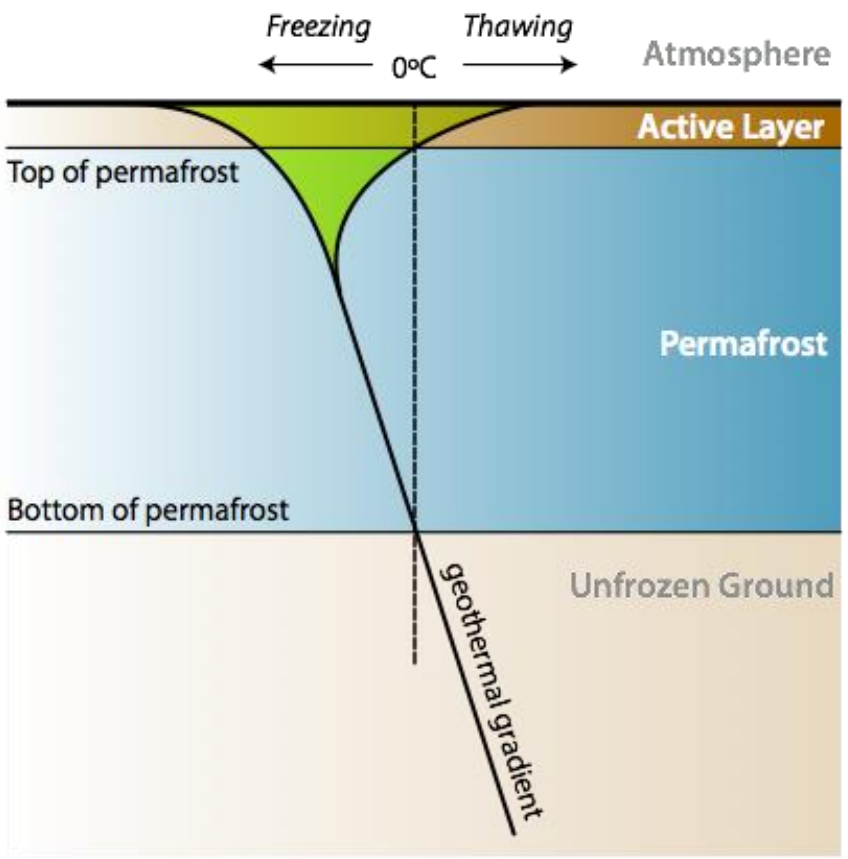
Vertical temperature profile in ground



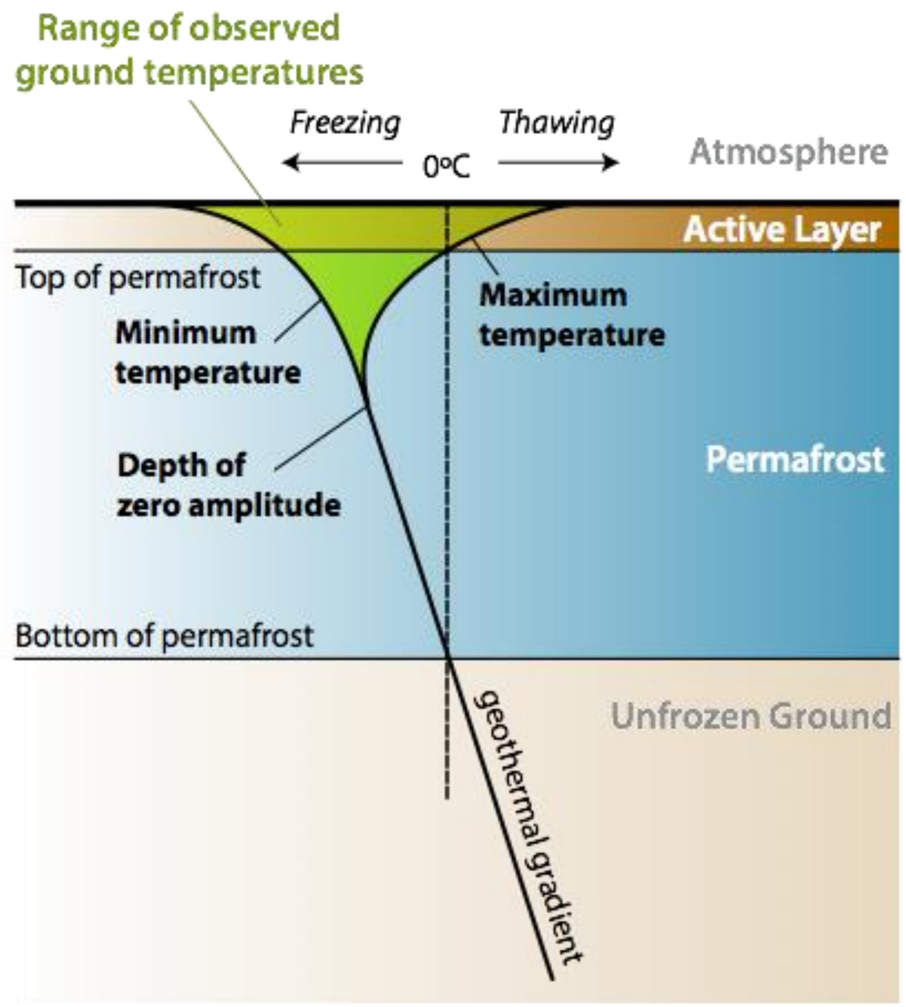
Vertical temperature profile in ground - permafrost



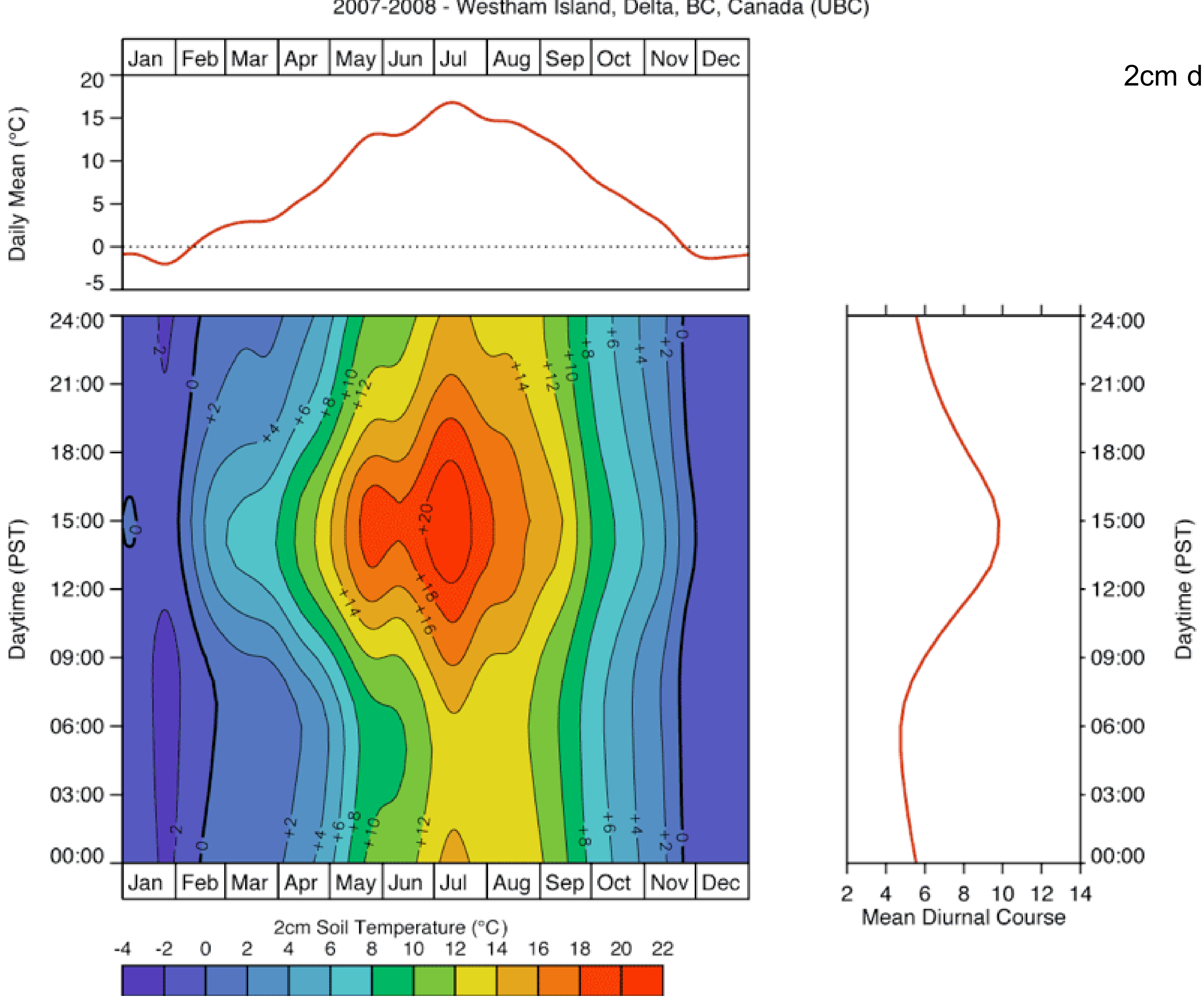
Vertical temperature profile in ground - permafrost



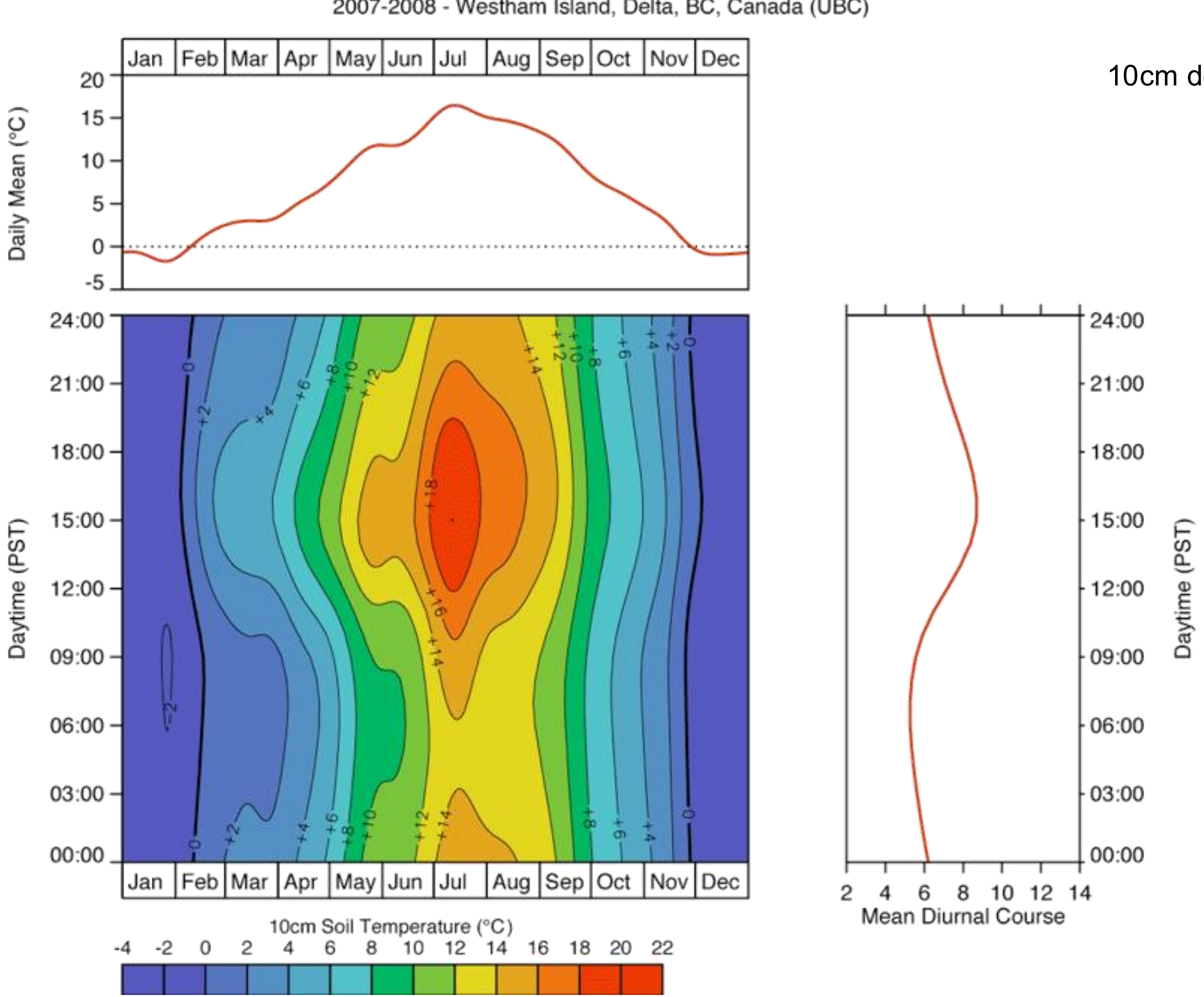
Vertical temperature profile in ground - permafrost

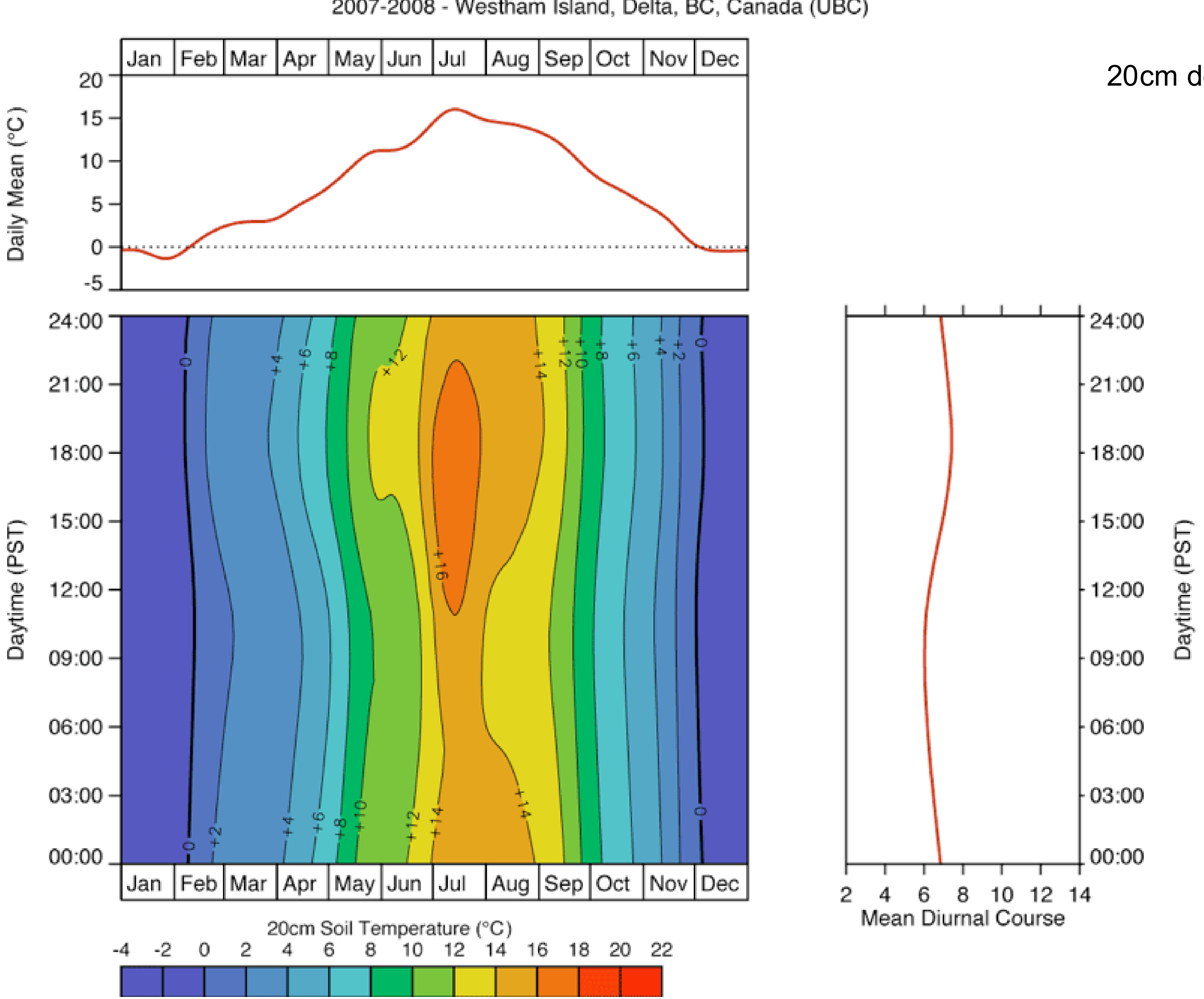


2cm depth


2007-2008 - Westham Island, Delta, BC, Canada (UBC)
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
Daily Mean (°C)
Daytime (PST)
2cm Soil Temperature (°C)
-4 -2 0 2 4 6 8 10 12 14 16 18 20 22
Mean Diurnal Course

10cm depth

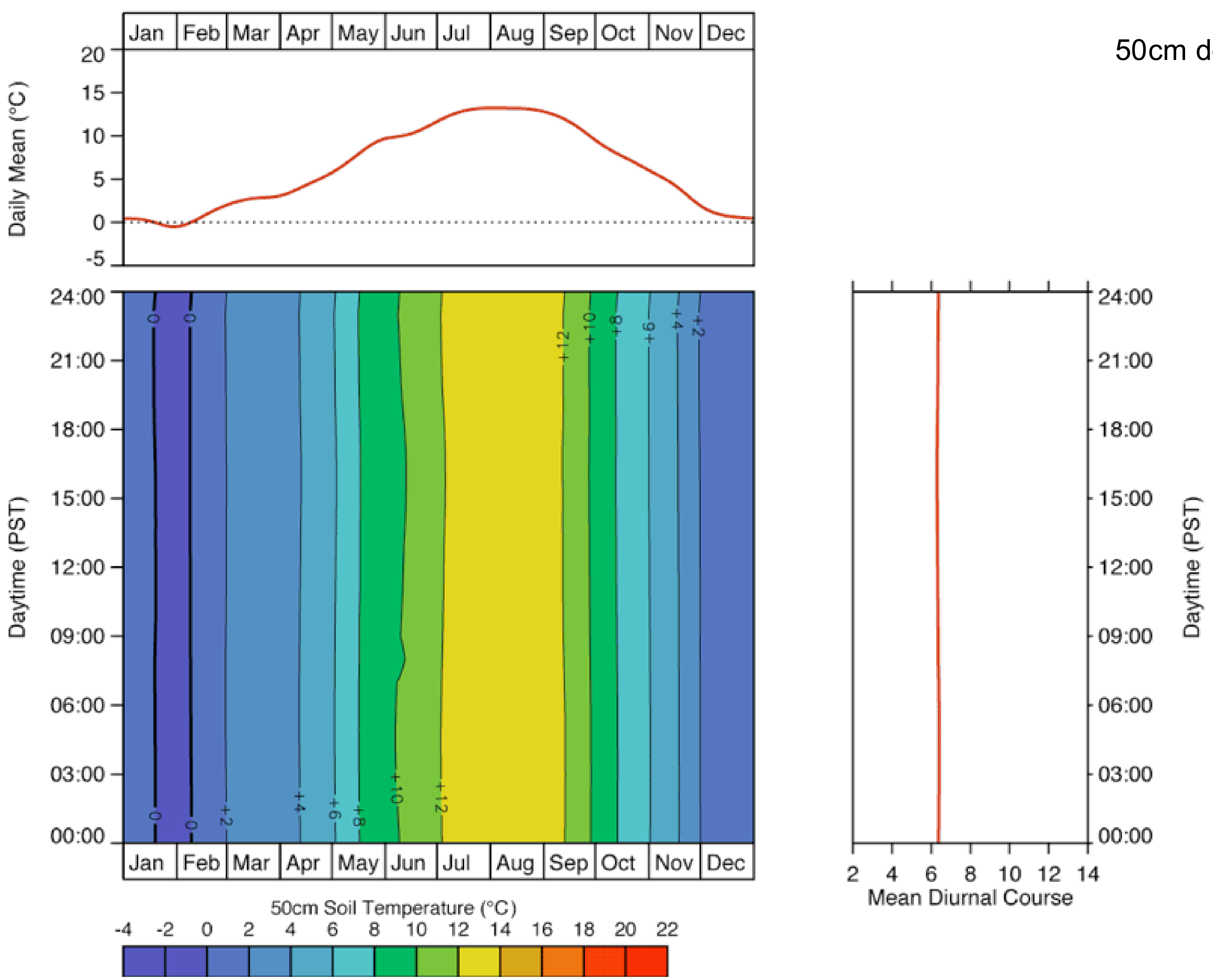

2007-2008 - Westham Island, Delta, BC, Canada (UBC)
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
Daily Mean (°C)
20 15 10 5 0 -5
Daytime (PST)
24:00 21:00 18:00 15:00 12:00 09:00 06:00 03:00 00:00
10cm Soil Temperature (°C)
-4 -2 0 2 4 6 8 10 12 14 16 18 20 22
2 4 6 8 10 12 14
Mean Diurnal Course


2007-2008 - Westham Island, Delta, BC, Canada (UBC)
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
Daily Mean (°C)
20 15 10 5 0 -5
Daytime (PST)
24:00 21:00 18:00 15:00 12:00 09:00 06:00 03:00 00:00
0 +2 +4 +6 +8 +10 +12 +14 +16
20cm Soil Temperature (°C)
-4 -2 0 2 4 6 8 10 12 14 16 18 20 22
2 4 6 8 10 12 14
Mean Diurnal Course

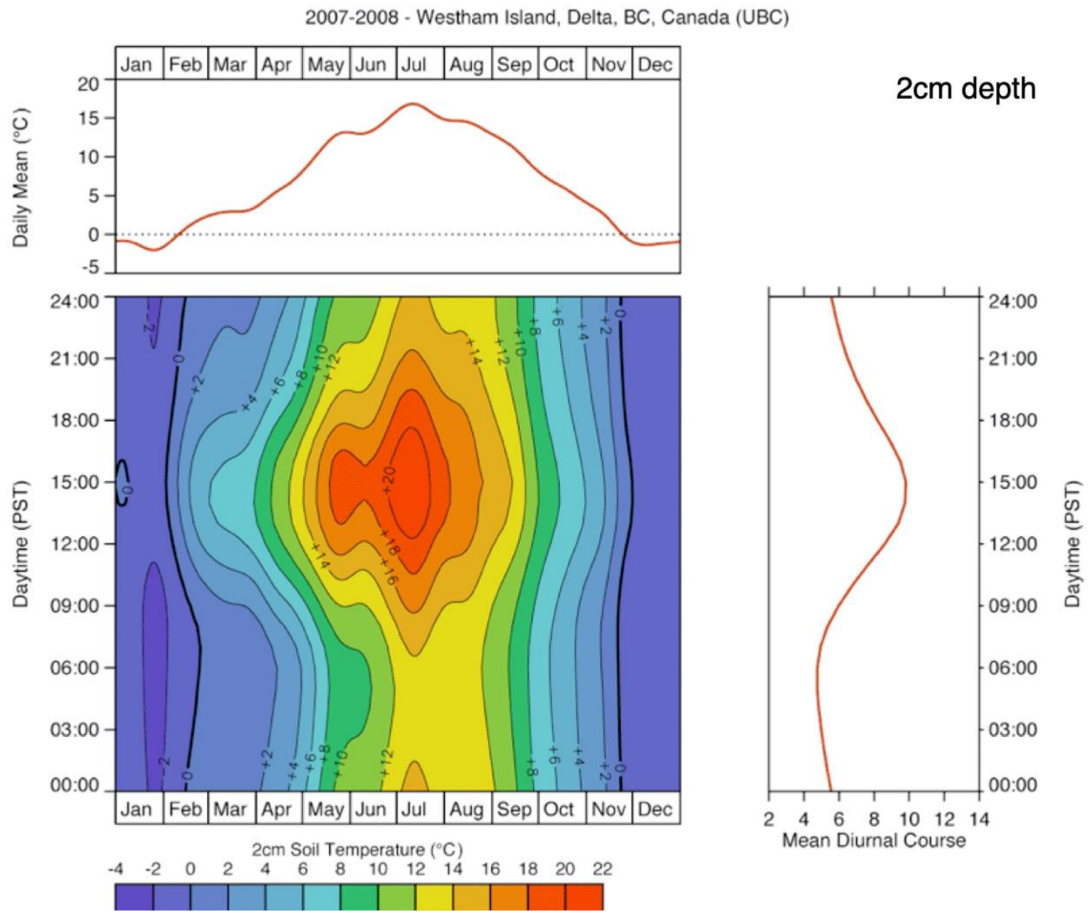
20cm depth

2007-2008 - Westham Island, Delta, BC, Canada (UBC)

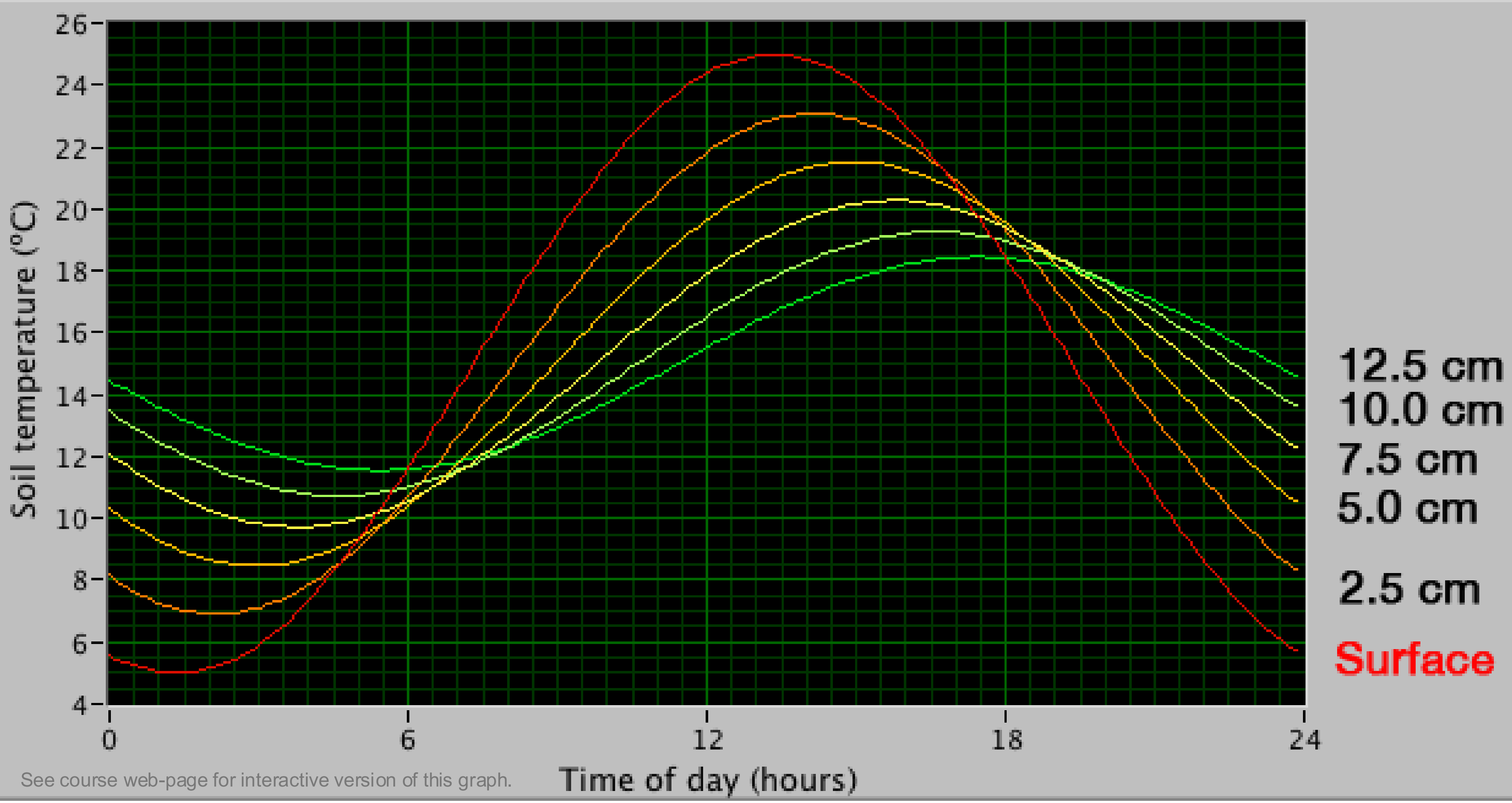
50cm depth



What are two things you notice about how the soil temperature wave changes with depth?



https://geog321.github.io/applets/stwave/



See course web-page for interactive version of this graph.

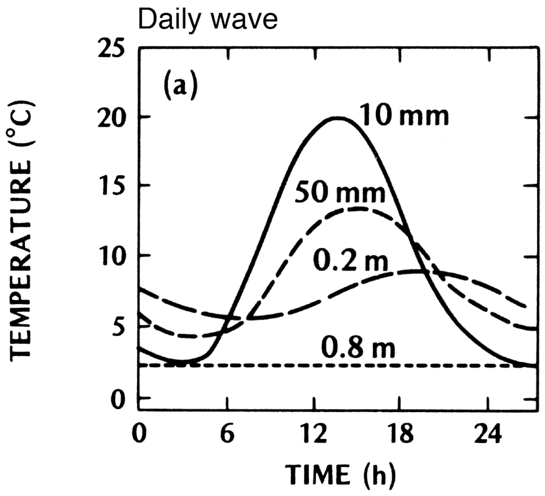
Soil temperature waves.

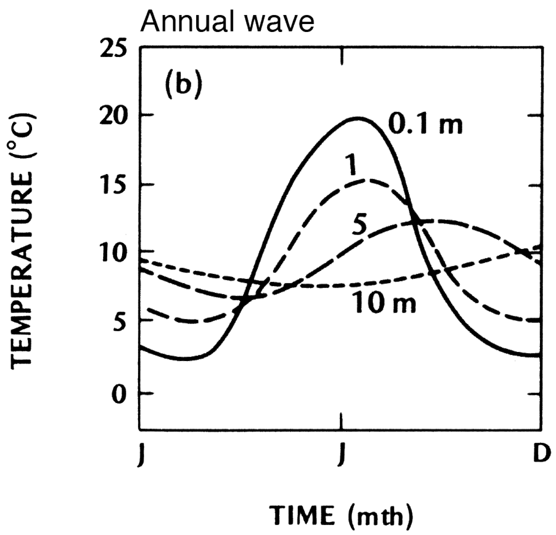
Surface wave moves down in response to radiative forcing at different periods P:

- annual (1 year) to ~14 m
- daily (1 day) to ~0.75 m
- cloud passage (~15 min) to ~0.1 m

Wave amplitude decreases with depth and there is a phase shift (time lag):

- **amplitude** decreases **exponentially**.
- **phase shift** (time lag) increases **linearly**.





Soil temperature waves.

The daily and annual periodic forcing by the sun creates a surface temperature (T_o) wave, which propagates into the soil below. If we assume

1) The **thermal diffusivity** $\kappa = k/C$ is **uniform**

2) Cycles are **sinusoidal** (better for year than day)

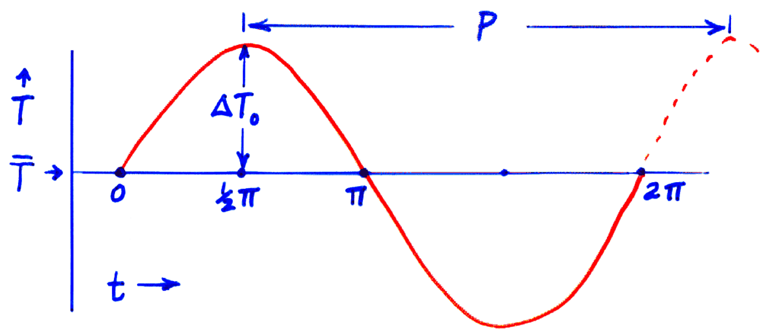
then the variation of T_o is given:

amplitude (1/2 range) of the (daily or annual) T_o wave

$$T_{(o,t)} = \overline{T_o} + \Delta T_o \sin \omega t$$

mean (daily or annual) surface temperature

angular **frequency** of oscillation $2\pi / P$ where P period



Calculate the angular frequency (ω) for a diurnal and annual temperature wave.

Calculate the angular frequency (ω) for a diurnal and annual temperature wave.

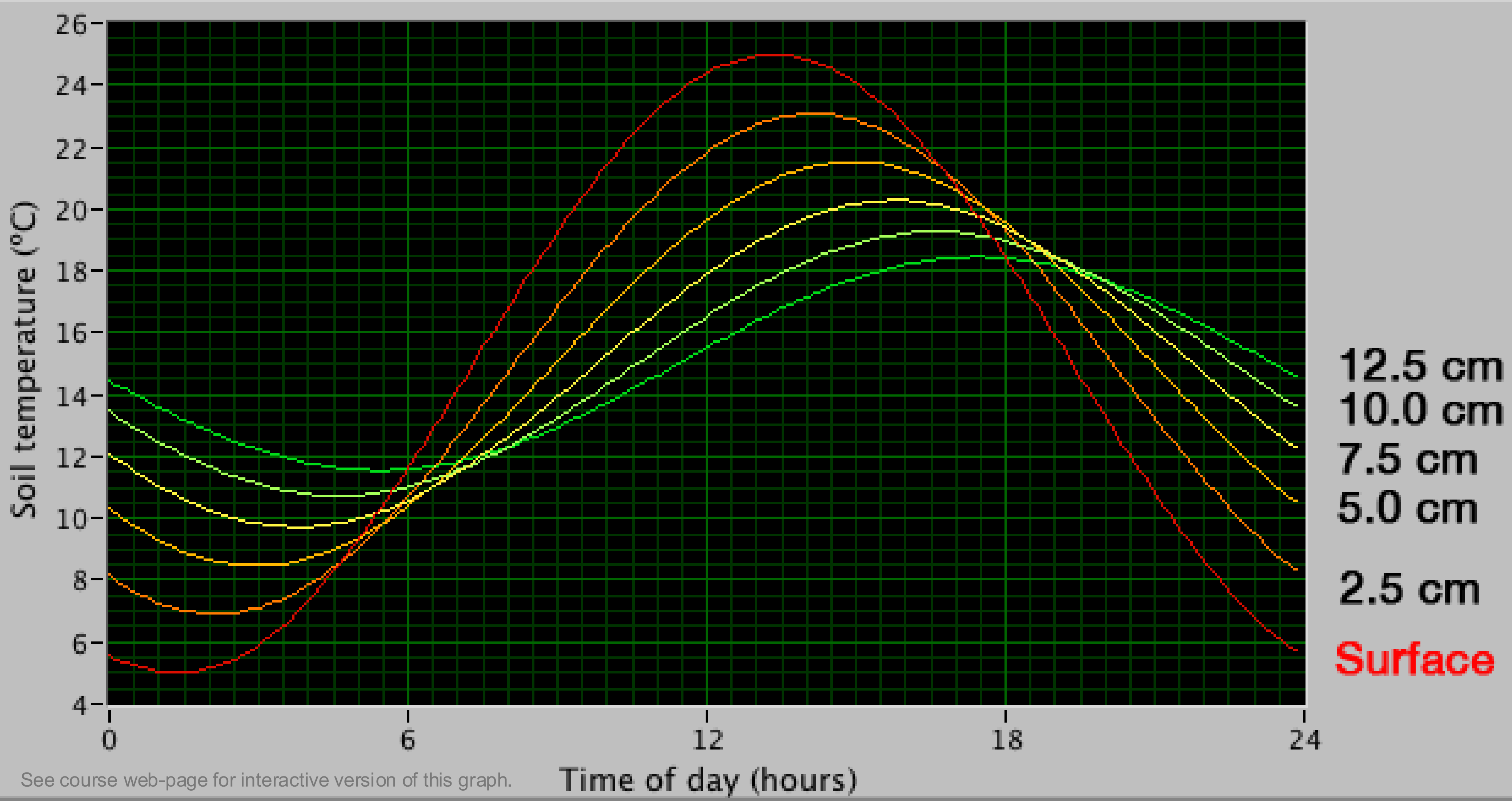
Diurnal:

$$\omega_d = \frac{2\pi}{P} = \frac{2\pi}{60 \times 60 \times 24\,\mathrm{s}} = \underline{7.27 \times 10^{-5}\,\mathrm{s}^{-1}}$$

Annual

$$\omega_a = \frac{2\pi}{P} = \frac{2\pi}{60 \times 60 \times 24 \times 365.25\,\mathrm{s}} = \underline{1.99 \times 10^{-7}\,\mathrm{s}^{-1}}$$

Analytical solution for sinusoidal forcing.



See course web-page for interactive version of this graph.

A solution of Fourier's heat conduction.

With the surface temperature wave equation as the boundary condition, we can find an analytic solution to the Fourier heat conduction equation in 1-D:

$$T_{(z,t)} = \overline{T_o} + \Delta T_o e^{-z(\omega/2\kappa)^{1/2}} \sin\left[\omega t - \left(\frac{\omega}{2\kappa}\right)^{1/2} z\right]$$

Wave **amplitude** at depth *z* relative to the surface temperature wave

Phase shift of the wave at depth *z*

Note, the wave decays exponentially with depth, and the decay is less in soils with large κ, or if the period is longer.

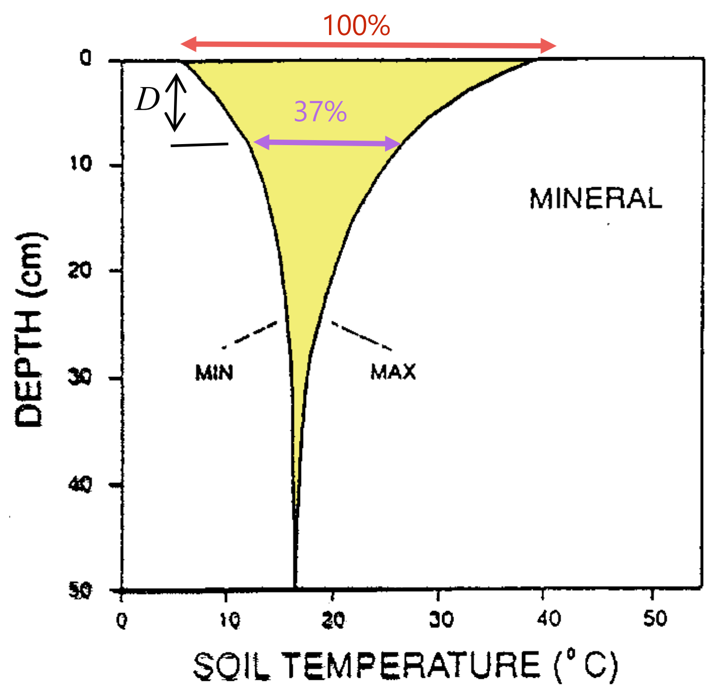
Damping depth.

The inverse of the exponent term defines a useful feature, the **damping depth**, D.

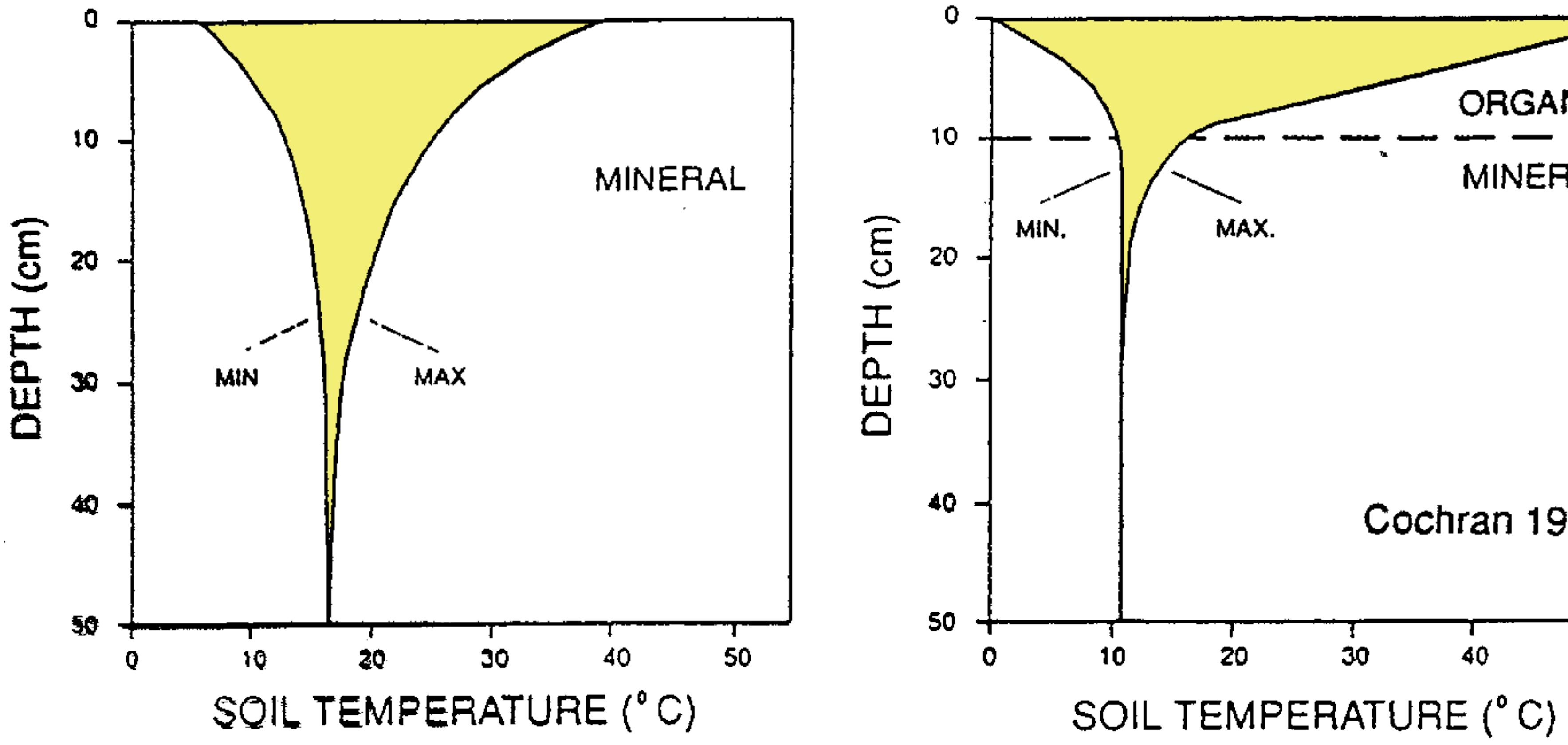
$$D = \sqrt{\frac{2\kappa}{\omega}} = \sqrt{\frac{\kappa P}{\pi}} \quad \star$$

It is the depth at which the surface temperature wave is reduced to e^{-1} (37%) of its value at $z = 0$.

At *3D* it drops below 5%, and *4.6 D* below 1%.



Example - effect of an organic cover.



Phase shift - maximum and minimum soil temperatures.

Term 2 of the 1-D analytical solution also includes the inverse of the damping depth and gives the **phase shift equation** for features such as maximum and minimum soil temperatures, i.e. which occurs at 0.5 π and 1.5 π.

$$\sin\left[\omega t - \left(\frac{\omega}{2\kappa}\right)^{1/2} z\right] = \pm 1$$

solving for *t*:

$$\Delta t_m = t_{m_2} - t_{m_1} = (z_2 - z_1)\left(\frac{1}{2\omega\kappa}\right)^{1/2}$$

where t_{m1} and t_{m2} are times at which maximum or minimum soil temperatures reach at z_1 and z_2 respectively, so Δt_m is the **phase shift**.

Calculate when the soil temperature reaches its maximum at 5 cm if the maximum surface temperature is measured at 13:00? Assume a thermal diffusivity κ = 5.0×10^{-7} m^{2} s^{-1}.

$$\Delta t_m = t_{m_2} - t_{m_1} = (z_2 - z_1)\left(\frac{1}{2\omega_d \kappa}\right)^{1/2}$$

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If the temperature maxima occurred at 13:00 at the surface, then the temperature maximum at 5 cm depth will occur at 13:00 + Δt_m:

$$\begin{aligned}\Delta t_m &= (0.05\,\mathrm{m} - 0\,\mathrm{m}) \times \left(\frac{1}{2 \times 7.27 \times 10^{-5}\,\mathrm{s}^{-1} \times 5 \times 10^{-7}\,\mathrm{m}^2\,\mathrm{s}^{-1}}\right)^{1/2} \\ &= 5864\,\mathrm{s} = 1.6\,\mathrm{h}\end{aligned}$$

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So the temperature maximum at 5 cm depth will occur at 13:00 + 01:38 = 14:38.

Phase shift - implications.

The phase shift at a certain depth is least in soils with large κ_S.

These idealized patterns are disrupted by:

- Variable cloud cover.
- Rain (percolation gives wetting front).
- Freezing/melting (latent heat effects).
- Snow cover (insulates).
- Soil inhomogeneities in the horizontal and vertical.

Take home points

- We observe **temperature waves** with annual, diurnal and even short-term (cloud-passage) periods.
- In a uniform soil, the wave **amplitude** decays **exponentially**, whereas the **phase-shift** is **linear** with depth.
- The **damping depth** is a commonly used term to quantify the depth to which 37% of the surface amplitude reaches down.