GEOG 321 - Reading Package Lectures 21, 22, 23, 25 & 26

SENSIBLE HEAT FLUX

In the turbulent surface layer the flux of sensible heat is given by:

$$Q_H = -C_a\,K_H\left(\frac{\partial\overline{T}}{\partial z} + \Gamma\right) = -C_a\,K_H\,\frac{\partial\overline{\theta}}{\partial z} \qquad (9.1)$$

where, K_H eddy conductivity (m^2 s^{-1}). The direction of the heat transfer (sign of Q_H) is determined by the sign of the temperature gradient. By day the gradient is negative (lapse) and Q_H is positive (i.e. directed from the surface into the lower atmosphere). By night the gradient is positive (inversion) and Q_H is negative. The adiabatic lapse rate Γ (see GEOB 200 and GEOB 201) is included to correct the observed temperature gradient for the effects of vertical atmospheric pressure changes. It is only of importance if the height interval is large (e.g. greater than 2 m). The use of potential temperature (θ) incorporates these effects.

The vertical transfer of sensible heat by eddies can be visualized with the aid of Figure 1. This shows the variation of vertical velocity (w), air temperature (T) and the associated instantaneous flux of sensible heat Q_H over a period of 5 min. The data are from a daytime unstable period in summer. The simultaneous record of air temperature are in phase with those of the vertical wind. Thus in unstable conditions an updraft (positive w) is associated with an increase of T, and downdraft (negative w) with a decrease of T, relative to its mean value. This occurs because unstable conditions are associated with a lapse T profile, and an updraft through the measurement level has originated closer to the ground where it is warmer. Conversely a downdraft comes from higher levels where it is cooler. For both situations (up- and downdraft) the net sensible heat transfer is therefore upwards. The instantaneous heat flux (lowest trace in Figure 1) also shows that most of the transfer tends to occur in ‘bursts’ coinciding with the upward movement of a buoyant thermal. The heat flux (Q_H) given by equation 9.1 should correspond to the time-average of the instantaneous flux.

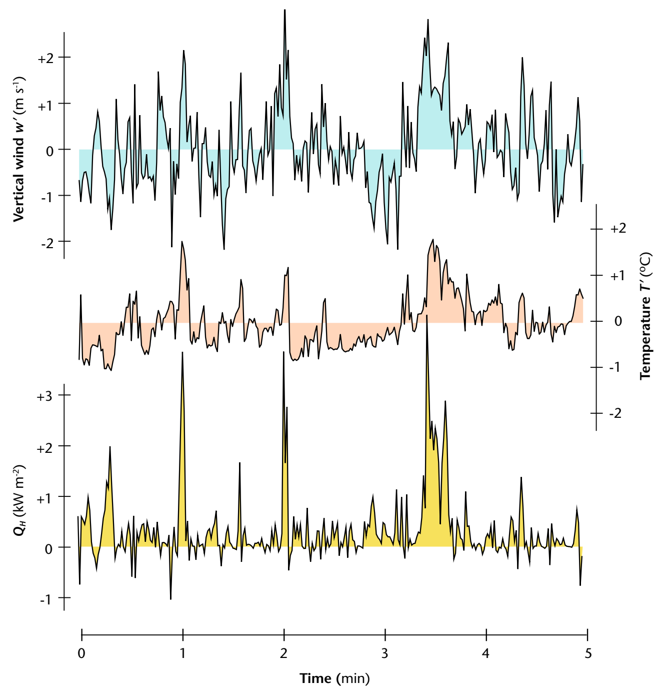

Figure 1: The relationships between vertical velocity (w') and air temperature (T') fluctuations, and the instantaneous sensible heat flux (Q_H). The time traces have been measured over 5 minutes on July 1, 2009, 14:00 on a 30m tower at Vancouver-Sunset. Note that Q_H is proportional to the product of the two traces $w'T'$.

PROFILE METHODS

There are two basic profile methods to evaluate turbulent fluxes: the aerodynamic (discussed here) and the Bowen ratio-energy balance (see lecture 33).

Both rely on the *Reynolds analogy*. This holds that the diffusion coefficients for momentum, heat, water vapour and carbon dioxide are equivalent, i.e.:

$$K_M = K_H = K_V = K_C \qquad (9.2)$$

and by extension to the aerodynamic resistances:

$$r_{aM} = r_{aH} = r_{aV} = r_{aC} \qquad (9.3)$$

This may be interpreted to mean that an eddy is non-discriminatory with respect to the property being transported. It will carry momentum, heat, vapour or carbon dioxide with equal facility. Thus determination of one K or r determines them all. Further, if we take ratios of turbulent flux density equations the Ks and rs cancel. For example, we find:

$$\frac{Q_H}{\tau} = \frac{C_a\,\Delta\overline{\theta}}{-\rho\,\Delta\overline{u}} = \frac{c_p\,\Delta\overline{\theta}}{-\Delta\overline{u}} \tag{9.4}$$

and

$$\frac{Q_E}{\tau} = \frac{L_v\,\Delta\overline{\rho_v}}{-\rho\,\Delta\overline{u}} \tag{9.5}$$

and

$$\frac{Q_H}{Q_E} = \frac{C_a\,\Delta\overline{\theta}}{L_v\,\Delta\overline{\rho_v}} \tag{9.6}$$

This is advantageous because measurement of two appropriate property differences, over the same height interval, and the knowledge of one flux enables the other flux to be obtained. For example, in the case of the first ratio listed a measure of τ plus the differences $\Delta\overline{\rho_v}$ and $\Delta\overline{u}$ allows determination of Q_E. Alternatively even if equality of the Ks cannot be assumed a knowledge of the behaviour of their ratios (e.g. K_V/K_M, etc.) allows the fluxes to be evaluated.

Aerodynamic approach. It should be pointed out that the unmodified aerodynamic method only applies under the following restricted conditions:

1. Neutral stability - buoyancy effects are absent.
2. Steady stateno marked shifts in the radiation or wind fields during the observation period.
3. Constancy of fluxes with heightno vertical divergence or convergence.
4. Similarity of all transfer coefficients.

Under these conditions the logarithmic wind profile (Reading Package Lecture 20) is valid, and the wind gradient ($\partial\overline{u}/\partial z$) is found to be inversely proportional to the height above the surface (z). Since the constant of proportionality can be equated to the slope of the neutral wind profile (i.e.k/u_*) it follows that:

$$u_* = k\,z\,(\partial\overline{u}/\partial z) \tag{9.7}$$

Writing the neutral wind profile equation separately for two heights z_1 and z_2, and subtracting the lower from the higher, we see that the friction velocity u_* can be found as:

$$u_* = \frac{k(\overline{u_2} - \overline{u_1})}{\ln z_2 - \ln z_1} = \frac{k\Delta\overline{u}}{\ln(z_2/z_1)} \tag{9.8}$$

Since $u_* = \sqrt{\tau/\rho}$ it follows that:

$$\tau = \rho\,k^2\left(\frac{\Delta\overline{u}}{\ln(z_2/z_1)}\right)^2 \tag{9.9}$$

and from Reynolds analogy:

$$Q_H = -C_a k^2 \frac{\Delta\overline{u}\,\Delta\overline{\theta}}{\left[\ln(z_2/z_1)\right]^2} \tag{9.10}$$

$$Q_E = -L_v k^2 \frac{\Delta\overline{u}\,\Delta\overline{\rho_v}}{\left[\ln(z_2/z_1)\right]^2} \tag{9.11}$$

and:

$$F_C = -L_v k^2 \frac{\Delta\overline{u}\,\Delta\overline{\rho_c}}{\left[\ln(z_2/z_1)\right]^2} \tag{9.12}$$

These are the neutral stability aerodynamic equations.

There are two major limitations to the use of this approach deriving from the necessary assumptions of neutral stability, and similarity of all coefficients. The former restricts its use to a very narrow range of natural conditions, and to periods when fluxes are likely to be small. However, even given a means to extend these equations for use in non-neutral stability, there is concern that the similarity principle does not apply (especially with regard to K_M).

Adjustments for diabatic situations. There is an extensive literature concerned with attempts to extend the aerodynamic method by incorporating adjustments which depend upon stability and which include empirical terms to account for non-similarity of the diffusion coefficients. Here we will review one simple approach. The Richardson Gradient Number (*Ri*, see Lecture 25) is a convenient means of categorizing dynamic stability (and the state of turbulence) in the lowest layers:

$$Ri = \frac{g}{\overline{\theta_v}}\,\frac{(\Delta\overline{\theta}/\Delta z)}{(\Delta\overline{u}/\Delta z)^2} \tag{9.13}$$

where g - acceleration due to gravity (m s^{-2}), θ_v - mean virtual potential temperature[1] in the layer Δz (K), and *Ri* is a dimensionless number. In general terms equation 9.13 shows that *Ri* relates the relative roles of buoyancy (numerator) to mechanical (denominator) forces (i.e. thermal to mechanical convection) in turbulent

[1] This is a 'corrected' temperature that includes density effects of water vapour. For the purpose of this course you can set this simply equal to the absolute temperature $\overline{T}$ in the layer Δz. The error is small. Atmospheric science students refer to advanced courses for the description of θ_v.

flow. Thus in strong lapse (unstable) conditions the free forces dominate and *Ri* is a negative number which increases with the size of the temperature gradient but is reduced by an increase in the wind speed gradient. In an inversion (stable) *Ri* is positive, and in neutral conditions *Ri* approaches zero.

The neutral form of the aerodynamic equations can be generalized according to stability (as given by *Ri*) in the following manner. From equation 9.7 rearrangement gives the neutral wind gradient as:

$$\frac{\partial \overline{u}}{\partial z} = \frac{u_*}{k\,z} \tag{9.14}$$

and in the general case we may write:

$$\frac{\partial \overline{u}}{\partial z} = \frac{u_*}{k\,z}\Phi_M \tag{9.15}$$

where, Φ_M - dimensionless stability function to account for curvature of the logarithmic wind profile due to buoyancy effects (Figure 2 in reading package for Lectures 20 / 22). The value of Φ_M is unity in the neutral case (i.e. equation 9.15 collapses to 9.14), and is greater or less than unity in stable and unstable conditions respectively. Similarly the neutral temperature, humidity and carbon dioxide gradients can be generalized to read:

$$\frac{\partial \overline{\theta}}{\partial z} = \frac{Q_H}{C_a\,k\,u_*\,z}\Phi_H \tag{9.16}$$

and

$$\frac{\partial \overline{\rho_v}}{\partial z} = \frac{Q_E}{L_v\,k\,u_*\,z}\Phi_V \tag{9.17}$$

and

$$\frac{\partial \overline{\rho_c}}{\partial z} = \frac{F_C}{k\,u_*\,z}\Phi_C \tag{9.18}$$

where Φ_H, Φ_V and Φ_C are dimensionless stability functions for heat, water vapour and carbon-dixode, Hence the operational equations 9.10 to 9.12 become:

$$Q_H = -C_a k^2 \frac{\Delta\overline{u}\,\Delta\overline{\theta}}{[\ln(z_2/z_1)]^2}(\Phi_M\Phi_H)^{-1} \tag{9.19}$$

$$Q_E = -L_v k^2 \frac{\Delta\overline{u}\,\Delta\overline{\rho_v}}{[\ln(z_2/z_1)]^2}(\Phi_M\Phi_V)^{-1} \tag{9.20}$$

and:

$$F_C = -L_v k^2 \frac{\Delta\overline{u}\,\Delta\overline{\rho_c}}{[\ln(z_2/z_1)]^2}(\Phi_M\Phi_C)^{-1} \tag{9.21}$$

Observations suggest that $\Phi_H = \Phi_V = \Phi_C = \Phi_M$ under moderately stable conditions, but that $\Phi_H = \Phi_V = \Phi_C = \Phi_M^2$ in the unstable case. Further empirical evidence leads to the following description of the stability functions used in equations 9.19 to 9.21:

Stable case (*Ri* positive)

$$(\Phi_M\Phi_x)^{-1} = (1 - 5Ri)^2 \tag{9.22}$$

Unstable case (*Ri* negative):

$$(\Phi_M\Phi_x)^{-1} = (1 - 16Ri)^{3/4} \tag{9.23}$$

where Φ_x is the appropriate stability function for the property being transferred. These relationships are plotted on logarithmic co-ordinates in Figure 2, which also shows the types of flow regimes existing under different stability conditions. When the atmosphere is neutral (*Ri* between ±0.01) thermal effects are minimal and only mechanical convection is present. Moving towards greater instability (to the right) thermal effects grow in importance through the mixed regime, and at values of *Ri* larger than -1.0 only thermal convection is in operation (weak horizontal motion, very strong convective instability). Conversely moving from neutrality towards greater stability (to the left) negative buoyancy increasingly dampens turbulent motion so that beyond *Ri* values of about +0.25 the flow is virtually laminar and vertical mixing is absent (weak horizontal motion, strong temperature inversion).

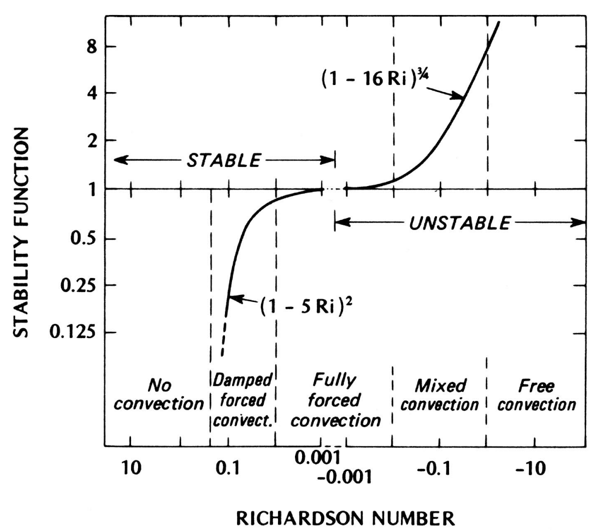

Figure 2: Non-dimensional 'stability function' $(\Phi_M\Phi_X)^{-1}$ plotted logarithmically against the Richardson number stability parameter. Fluxes calculated in non-neutral conditions using flux-gradient equations valid for neutral conditions must be multiplied by this factor. Also showing the characteristic flow regimes at different stabilities (after Thom, 1975).

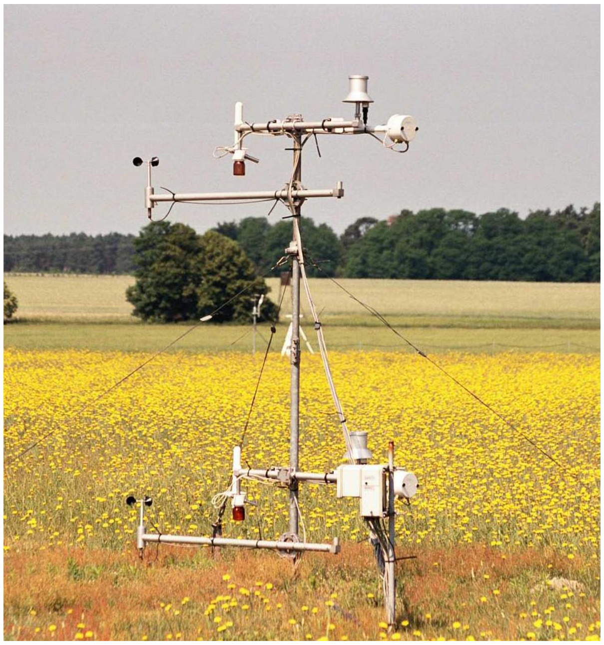

Figure 3: A meteorological mast to evaluate fluxes with two levels of wind $\overline{u}$, temperature $\overline{u}$ and absolute humidity $\overline{\rho_v}$ measurements over a crop in Falkenberg, Germany.

In summary, although the basic aerodynamic approach is only applicable in neutral conditions semi-empiric relationships can be used to extend its usefulness to a wide range of stability regimes. The evaluation of fluxes via this method requires the accurate measurement of a wind difference, and the difference of a related property (usually over the same height interval). Theoretically only two levels are required, but in practice this is open to error in a single instrument, so it is advisable to use a number of heights and average the differences. Should the height of the surface roughness elements necessitate the inclusion of a zero-plane displacement (z_d) in the wind profile, heights should be modified accordingly. Averaging periods of about 30 minutes are normally appropriate.

EDDY COVARIANCE METHOD

All atmospheric entities show short-period fluctuations about their longer term mean value. This is the result of turbulence which causes eddies to move continually around carrying with them their properties derived elsewhere. Recall from Lecture 19 that we can apply *Reynolds decomposition* and write that the value of an entity (s) consists of its mean value ($\overline{s}$), and a fluctuating part (s'), so that:

$$s = \overline{s} + s' \qquad \star \quad (9.24)$$

where the overbar indicates a time-averaged property and the prime signifies instantaneous deviation from the mean. The vertical wind trace in Figure 2 illustrates these two components: the horizontal line at w=0 is the value $\overline{w}$ since at an extensive site mass continuity requires that as much air moves up as moves down over a reasonable period of time (e.g. 10 min); and the detail of the fluctuating trace gives the value of w' at any instant as a positive or negative quantity depending upon whether it is above the mean (an updraft) or below it (a downdraft). The properties contained by, and therefore transported with, an eddy are its mass (which by considering unit volume is given by its density, (ρ), its vertical velocity (w) and the volumetric content of any entity it possesses (s). Since each one can be broken into a mean and a fluctuating part the mean vertical flux density of the entity (S) can therefore be written:

$$S = \overline{(\overline{\rho} + \rho')(\overline{w} + w')(\overline{s} + s')} \qquad (9.25)$$

which upon full expansion yields:

$$\begin{aligned} S \quad = \quad & \overline{\overline{\rho}\,\overline{w}\,\overline{s}} + \overline{\overline{\rho}\,\overline{w}\,s'} + \overline{\overline{\rho}\,w'\,\overline{s}} + \overline{\overline{\rho}\,w'\,s'} \\ & + \overline{\rho'\,\overline{w}\,\overline{s}} + \overline{\rho'\,\overline{w}\,s'} + \overline{\rho'\,w'\,\overline{s}} \\ & + \overline{\rho'\,w'\,s'} \end{aligned} \qquad (9.26)$$

Although equation 9.26 looks rather formidable it can be greatly simplified. First, all terms involving a single primed quantity are eliminated because by definition the average of all their fluctuations equals zero (i.e. we lose the second, third and fifth terms). Second, we may neglect terms involving fluctuations of ρ' since air density is considered to be virtually constant in the lower atmosphere (i.e. we lose the sixth, seventh and eighth terms. Third, if observations are restricted to uniform terrain without areas of preferred vertical motion (i.e. no 'hotspots' or standing waves) we may neglect terms containing the mean vertical velocity, as $\overline{w} = 0$ (i.e. we lose also the first term). With these assumptions equation 9.26 reduces to the form of the relation underlying the eddy fluctuation approach, viz.:

$$S = \rho\overline{w'\,s'} \qquad (9.27)$$

where the bar over the ρ has been dropped since it is considered to be a constant. At first glance it might appear as though this term also could be ignored since both w' and s' averaged over time will be zero. However, the overbar denotes the time average of the instantaneous covariances of w and s (i.e. the time average of their instantaneous product) and this will only rarely be negligible.

In terms of the fluxes and entities with which we are concerned, equation 9.27 can be written:

$$\tau = -\rho\overline{u'\,w'} \qquad \star \quad (9.28)$$

$$Q_H = C_a \overline{w' T'} \qquad \star \qquad (9.29)$$

$$Q_E = L_v \overline{w' \rho_v'} \qquad \star \qquad (9.30)$$

$$F_C = \overline{w' \rho_c'} \qquad \star \qquad (9.31)$$

and, Figure 2 very clearly illustrates how the product of w' and T' combine to produce an instantaneous sensible heat flux (Q_H). The time average of this heat flux is the value given by equation 9.29.

To obtain the fluxes given by these equations it is necessary to have instruments which can very rapidly sense virtually every variation in the vertical wind velocity and in the entity under study, and processing and recording equipment must be capable of integrating and/or quickly recording many readings. The response of the instruments should be matched, and sufficiently fast to sense the properties of the smallest eddies capable of contributing significantly to the transport. Since the size of eddies increases with height these requirements become increasingly harder to meet closer to the ground. Typical measurement frequencies in the surface layer are between 40 and 10 Hz depending on the proximity to the ground and the roughness of the surface.

Typical w sensors for eddy fluctuation instruments include hot-wire anemometers and ultrasonic anemometers (sonics). Temperature is measured by fine-wire resistance or thermocouple elements or acoustic thermometers, and both water vapour and carbon dioxide by infra-red gas analysers (IRGA).

The method has the great advantages of being based on an essentially simple theory; of measuring the fluxes directly; and of requiring no additional specifications of the nature of the surface (such as roughness) or of the atmosphere (such as stability).

If Q_H or Q_E are measured using the eddy covariance method, it is usually fairly straightforward to find the other. With additional measurements of Q^* (by net radiometer) and Q_G (by soil heat flux plate) the other turbulent flux density can be solved as the residual of the surface energy balance equation.

Figure 4 shows instruments capable of measuring turbulent fluxes via the eddy fluctuation approach. The ultrasonic anemometer measured vertical wind at 40 Hz (w'), the fine-wire thermocouple (T') combination can give Q_H. The anemometer plus the infra-red gas analyzer (ρ_v and ρ_c) which can give Q_E (or E) and F_C.

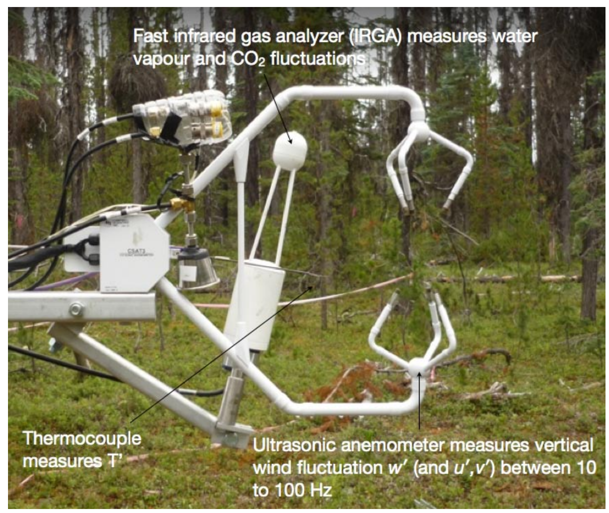

Figure 4: A typical eddy covariance system to directly measure turbulent fluxes of heat (Q_H), latent heat (Q_E) and fluxes of carbon dioxide F_C.