# GEOG 321 - Reading Package Lectures 18

Stull R. B., 2000: ‘Meteorology for Scientists and Engineers’,
Brooks / Cole, Pages 82-84, 87

# TURBULENCE

## Mean and Turbulent Parts

Wind can be quite variable. The total wind speed is the superposition of three types of flow:

**mean wind** – relatively constant, but varying slowly over the course of hours

**waves** – regular (linear) oscillations of wind, often with periods of ten minutes or longer

**turbulence** – irregular, quasi-random, nonlinear variations or gusts, with durations of seconds to minutes

These flows can occur individually, or in any combination. Waves are discussed in Chapters 7 and 10. Here, we focus on mean wind and turbulence.

Let $U(t)$ be the x-direction component of wind at some instant in time, $t$. Different values of $U(t)$ can occur at different times, if the wind is variable. By averaging the **instantaneous wind** measurements over a time period, $P$, we can define a **mean wind** $\overline{U}$, where the overbar denotes an average. This mean wind can be subtracted from the instantaneous wind to give the **turbulence** or gust part (Fig 4.23).

Similar definitions exist for the other wind components and temperature and humidity:

$$u'(t) = U(t) - \overline{U} \quad (4.20a)$$

$$v'(t) = V(t) - \overline{V} \quad (4.20b)$$

$$w'(t) = W(t) - \overline{W} \quad (4.20c)$$

$$T'(t) = T(t) - \overline{T} \quad (4.20d)$$

$$r'(t) = r(t) - \overline{r} \quad (4.20e)$$

Thus, the wind can be considered as a sum of mean and turbulent parts.

The averages in eq. (4.20) are defined over time or over horizontal distance. For example, the mean temperature is the sum of all individual temperature measurements, divided by the total number $N$ of data points:

$$\overline{T} = \frac{1}{N}\sum_{k=1}^{N} T_k \quad \bullet(4.21)$$

where $k$ is the index of the data point (corresponding to different times or locations). The averaging time in eq. (4.21) is typically about 0.5 h. If you average over space, typical averaging distance is 50 to 100 km.

Short term fluctuations (described by the primed quantities) are associated with small-scale swirls of motion called **eddies**. The superposition of many such eddies of various scales makes up the **turbulence** that is imbedded in the mean flow.

Molecular viscosity in the air causes friction between the eddies, tending to reduce the turbulence intensity. Thus, turbulence is NOT a conserved quantity, but is **dissipative**. Turbulence decays and

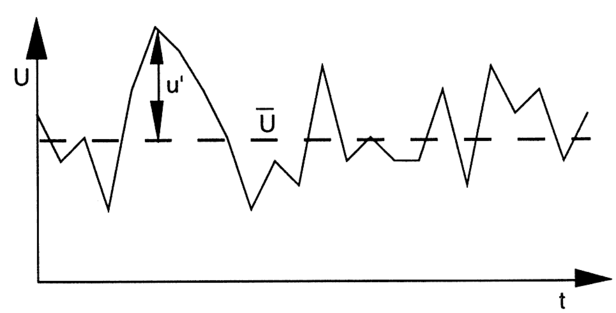


**Figure 4.23**
*The instantaneous wind speed U shown by the zigzag thin line. The average wind speed $\overline{U}$ is shown by the thicker horizontal dashed line. A gust velocity u' is the instantaneous deviation of the instantaneous wind from the average.*

**Solved Example**

Given the following measurements of total instantaneous temperature, $T$, find the average $\overline{T}$. Also, find the $T'$ values.

| t (min) | T (°C) | t (min) | T (°C) |
|---|---|---|---|
| 1 | 12 | 6 | 13 |
| 2 | 14 | 7 | 10 |
| 3 | 10 | 8 | 11 |
| 4 | 15 | 9 | 9 |
| 5 | 16 | 10 | 10 |

**Solution**

As specified by eq. (4.21), adding the ten temperature values and dividing by ten gives the average $\overline{T}$ = **12.0°C**. Subtracting this average from each instantaneous temperature gives:

| t (min) | T '(°C) | t (min) | T '(°C) |
|---|---|---|---|
| 1 | 0 | 6 | 1 |
| 2 | 2 | 7 | –2 |
| 3 | –2 | 8 | –1 |
| 4 | 3 | 9 | –3 |
| 5 | 4 | 10 | –2 |

**Check**: The average of these $T'$ values should be zero, by definition. In fact, this is a good way to check for mistakes.

**Discussion**: If a positive $T'$ corresponds to a positive $w'$, then warm air is moving up. This contributes positively to the heat flux.

disappears unless there are active processes to generate it. Two such production processes are **convection**, associated with warm air rising and cool air sinking, and **wind shear**, the change of wind speed or direction with height.

Normally, weather forecasts are made for mean conditions, not turbulence. Nevertheless, the net effects of turbulence on mean flow must be included. Idealized average turbulence effects are given in Chapts. 2, 6, and 9 for the heat, moisture, and momentum budgets, respectively. Appendix H describes turbulence parameterization in detail.

Meteorologists use statistics to quantify the net effect of turbulence. Some statistics are described next. In this chapter we will continue to use the overbar to denote the mean conditions. However, we drop the overbar in subsequent chapters to simplify the notation.

## Variance and Standard Deviation

The **variance** $\sigma^2$ of vertical velocity is an overall statistic of gustiness:

$$\sigma_w^{\ 2} = \frac{1}{N}\sum_{k=1}^{N}(W_k - \overline{W})^2 = \frac{1}{N}\sum_{k=1}^{N}(w_k{}')^2 = \overline{w'^{\ 2}} \qquad \bullet(4.22)$$

Similar definitions can be made for $\sigma_u^{\ 2}$, $\sigma_v^{\ 2}$, $\sigma_\theta^{\ 2}$, etc. Statistically, these are called "biased" variances. Velocity variances can exist in all three directions, even if there is a mean wind in only one direction.

The **standard deviation** $\sigma$ is defined as the square-root of the variance, and can be interpreted as an average gust (for velocity), or an average turbulent perturbation (for temperatures and humidities, etc.). For example, standard deviations for vertical velocity, $\sigma_w$, and potential temperature, $\sigma_\theta$, are:

$$\sigma_w = \sqrt{\sigma_w^{\ 2}} = \overline{(w')^2}^{\,1/2} \qquad (4.23a)$$

$$\sigma_\theta = \sqrt{\sigma_\theta^{\ 2}} = \overline{(\theta')^2}^{\,1/2} \qquad (4.23b)$$

Larger variance or standard deviation of velocity means more intense turbulence.

**Science Graffito**

"Big whirls have little whirls that feed on their velocity, And little whirls have lesser whirls and so on to viscosity – in the molecular sense." L.F. Richardson, 1922.

## Isotropy

If turbulence has the same variance in all three directions, then turbulence is said to be **isotropic**. Namely:

$$\sigma_u{}^2 = \sigma_v{}^2 = \sigma_w{}^2 \qquad \bullet(4.27)$$

Don't confuse this word with "isentropic", which means adiabatic or constant entropy.

Turbulence is **anisotropic** (not isotropic) in many situations. During the daytime over bare land, rising thermals create stronger vertical motions than horizontal. Hence, a smoke puff becomes **dispersed** (i.e., spread out) in the vertical faster than in the horizontal. At night, vertical motions are very weak, while horizontal motions can be larger. This causes smoke puffs to fan out horizontally with only little vertical dispersion in statically stable air.

**Solved Example (§)**

(a) Given the following V-wind measurements. Find the mean wind speed, and standard deviation.
(b) If the vertical standard deviation is 1 m/s, is the flow isotropic?

| $t$ (h) | $V$ (m/s) |
|---|---|
| 0.1 | 2 |
| 0.2 | –1 |
| 0.3 | 1 |
| 0.4 | 1 |
| 0.5 | –3 |
| 0.6 | –2 |
| 0.7 | 0 |
| 0.8 | 2 |
| 0.9 | –1 |
| 1.0 | 1 |

**Solution**

Given: Velocities listed above, $\sigma_w = 1$ m/s.
Find: $\overline{V}$ = ? m/s, $\sigma_v$ = ? m/s, isotropy = ?

(a) Use eq. (4.21), except for $V$ instead of $T$:

$$\overline{V}(z) = \frac{1}{n}\sum_{i=1}^{n} V_i(z) = \frac{1}{10}(0) = \underline{\mathbf{0\ m/s}}$$

Use eq. (4.22), but for $V$: $\sigma_v{}^2 = \frac{1}{n}\sum_{i=1}^{n}(V_i - \overline{V})^2$

$= (0.1)\cdot(4+1+1+1+9+4+0+4+1+1) = 2.6\ \mathrm{m^2/s^2}$

$\sigma_v = \sqrt{2.6 m^2 \cdot s^{-2}}$ = **<u>1.61 m/s</u>**

**Check**: Units OK. Physics OK.
**Discussion**: (b) **Anisotropic**, because $\sigma_v > \sigma_w$.

## Turbulence Kinetic Energy

An overall measure of the intensity of turbulence is the turbulence kinetic energy per unit mass (*TKE*):

$$TKE = 0.5\cdot\left[\overline{(u')^2} + \overline{(v')^2} + \overline{(w')^2}\right] \qquad \bullet(4.28a)$$

$$TKE = 0.5\cdot\left[\sigma_u{}^2 + \sigma_v{}^2 + \sigma_w{}^2\right] \qquad \bullet(4.28b)$$

*TKE* is usually produced at the scale of the boundary-layer depth. The production is mechanically by wind shear and buoyantly by thermals.

Turbulent energy cascades through the **inertial subrange**, where the large size eddies drive medium ones, which in turn drive smaller eddies. Molecular viscosity continuously damps the tiniest (**microscale**) eddies, dissipating *TKE* into heat. *TKE* is not conserved.

## Turbulent Fluxes and Covariances

Rewrite eq. (4.20) for variance of $w$ as

$$\text{var}(w) \quad = \frac{1}{N}\sum_{k=1}^{N}(W_k - \overline{W})\cdot(W_k - \overline{W}) \qquad (4.35)$$

By analogy, a **covariance** between vertical velocity $w$ and potential temperature $\theta$ can be defined as:

$$\text{covar}(w,\theta) = \frac{1}{N}\sum_{k=1}^{N}(W_k - \overline{W})\cdot(\theta_k - \overline{\theta})$$

$$= \frac{1}{N}\sum_{k=1}^{N}(w_k{}')\cdot(\theta_k{}') \qquad \bullet(4.36)$$

$$= \overline{w'\theta'}$$

where the overbar still denotes an average. Namely, one over $N$ times the sum of $N$ terms (see middle line of eq. 4.36) is the average of those items. Comparing eqs. (4.35) with (4.36), we see that variance is just the covariance between a variable and itself.

Covariance indicates the amount of common variation between two variables. It is positive where both variables increase or decrease together. Covariance is negative for opposite variation, such as when one variable increases while the other decreases. Covariance is zero if one variable is unrelated to the variation of the other.

The **correlation coefficient** is defined as the covariance normalized by the standard deviations of the two variables. Using vertical velocity and potential temperature for illustration:

$$r_{w,\theta} \equiv \frac{\overline{w'\theta'}}{\sigma_w \cdot \sigma_\theta} \qquad \bullet(4.37)$$

By normalized, we mean that $-1 \le r \le 1$. A correlation coefficient of positive one indicates a perfect correlation (both variables increase or decrease together proportionally), negative one indicates perfect opposite correlation, and zero indicates no correlation. Because it is normalized, the correlation coefficient gives no information on the absolute magnitudes of the variations.

In the ABL, many turbulent variables are correlated. For example, in the statically unstable ABL (Fig 4.25a), parcels of warm air rise and while other cool parcels sink in convective circulations. Warm air ($\theta' = +$) going up ($w' = +$) gives a positive product ($w'\theta'_{up} = +$). Cool air ($\theta' = -$) going down* ($w' = -$) also gives a positive product ($w'\theta'_{down} = +$).

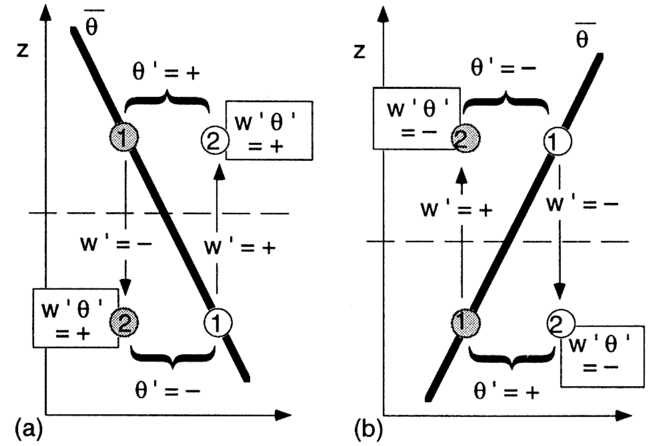


**Figure 4.25**
*(a) Relationship between turbulent potential temperature θ and vertical velocity w for a statically unstable environment (e.g., daytime). (b) Same, but for a statically stable environment (e.g., nighttime). In both figures, the thick line represents the ambient environment, circles represent air parcels, with white being the warm air parcel and gray being cool. Numbers 1 and 2 indicate starting and ending positions of each air parcel during time interval Δt.*

The average of those two products is also positive [ $\overline{w'\theta'} = 0.5\cdot(w'\theta'_{up} + w'\theta'_{down}) = +$ ]. The result gives positive correlation coefficients $r_{w\theta}$ during convection, which is typical during daytime.

Similarly, for statically stable conditions (Fig 4.25b) where wind-shear-induced turbulence drives vertical motions against the restoring buoyant forces (see Chapt. 7), one finds cold air moving up, and warm air moving down. (Parcel warmness or coldness is measured relative to the ambient mean potential temperature $\overline{\theta}$ at the same ending height as the parcel.) This gives $\overline{w'\theta'} = -$, which is often the case during night.